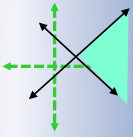
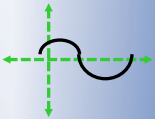


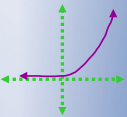
$$5x - 2y \leq 75$$



$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$



$$S = Pe^{rt}$$



$$APY = \left(1 + \frac{r}{n}\right)^n - 1$$

Math 1090 ~ Business Algebra

Section 2.4 Inverse Matrices

Objectives:

- Use Gauss-Jordan techniques to find an inverse of a matrix, if it exists.
- Use inverse matrices to solve systems of equations.

Inverse Matrix

A^{-1} , read "A inverse," is a matrix such that

- $A^{-1} \cdot A = I = A \cdot A^{-1}$ ($I =$ identity matrix that's same size as A)
- A^{-1} can only exist for a square matrix

Ex 1: Find A^{-1} for

$$\begin{aligned} \text{a) } A &= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \\ &\xrightarrow{(-3)} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array} \right] \\ &\xrightarrow{(-\frac{1}{2})} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -2 & -3 & 1 \end{array} \right] \\ &\xrightarrow{(-2)} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & \frac{3}{2} & \frac{1}{2} \\ 0 & -2 & -3 & 1 \end{array} \right] \end{aligned}$$

Method to find A^{-1} for A

- If A is not square, A^{-1} does not exist (DNE).
- If A is square,
 - Augment A with the identity matrix.
 - Perform elementary row operations on the augmented matrix until the left side is I , the identity matrix.
 - What is on the right side is A^{-1} .

$$\left[\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & 1 & \frac{3}{2} & \frac{1}{2} \end{array} \right] \Rightarrow A^{-1} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & \frac{1}{2} \end{bmatrix}$$

$$\text{b) } A = \begin{bmatrix} 5 & 4 \\ 0 & 2 \end{bmatrix}$$

$$\begin{aligned} A^{-1} &= \frac{1}{5(2) - 4(0)} \begin{bmatrix} 2 & -4 \\ 0 & 5 \end{bmatrix} \\ &= \frac{1}{10} \begin{bmatrix} 2 & -4 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & -\frac{2}{5} \\ 0 & \frac{1}{2} \end{bmatrix} \end{aligned}$$

check:

$$AA^{-1} = \begin{bmatrix} 5 & 4 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} \frac{1}{5} & -\frac{2}{5} \\ 0 & \frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 5(\frac{1}{5}) + 4(0) & 5(-\frac{2}{5}) + 4(\frac{1}{2}) \\ 0(\frac{1}{5}) + 2(0) & 0(-\frac{2}{5}) + 2(\frac{1}{2}) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \quad \checkmark$$

Formula for A^{-1}
of a 2×2 matrix
(if A^{-1} exists)

$$\text{If } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix},$$

$$\text{then } A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Ex 2: Find A^{-1} if possible.

a) $A = \begin{bmatrix} 1 & 3 & 5 & 7 \\ -5 & 1 & 0 & 1 \\ 3 & -2 & 7 & 0 \end{bmatrix}$

3x4 matrix which is NOT square

$\Rightarrow A^{-1}$ DNE

b) $A = \begin{bmatrix} 7 & -4 & 6 \\ 7 & -4 & 5 \\ 2 & -1 & 1 \end{bmatrix}$
 (3x3 matrix)

$$\begin{array}{l}
 \begin{array}{l}
 \text{(+3)} \rightarrow \left[\begin{array}{ccc|ccc}
 \textcircled{7} & -4 & 6 & 1 & 0 & 0 \\
 7 & -4 & 5 & 0 & 1 & 0 \\
 2 & -1 & 1 & 0 & 0 & 1
 \end{array} \right] \\
 \begin{array}{l}
 \text{(-7)} \rightarrow \left[\begin{array}{ccc|ccc}
 1 & -4 & 6 & 1 & 0 & 0 \\
 \textcircled{7} & -4 & 5 & 0 & 1 & 0 \\
 2 & -1 & 1 & 0 & 0 & 1
 \end{array} \right] \\
 \begin{array}{l}
 \text{(-2)} \rightarrow \left[\begin{array}{ccc|ccc}
 1 & -4 & 6 & 1 & 0 & 0 \\
 0 & 3 & -16 & -7 & 1 & 0 \\
 \textcircled{2} & -1 & 1 & 0 & 0 & 1
 \end{array} \right] \\
 \begin{array}{l}
 \text{(+3)} \rightarrow \left[\begin{array}{ccc|ccc}
 1 & -1 & 3 & 1 & 0 & -3 \\
 0 & 3 & -16 & -7 & 1 & 21 \\
 0 & 1 & -5 & -2 & 0 & 7
 \end{array} \right] \\
 \begin{array}{l}
 \text{(-3)} \rightarrow \left[\begin{array}{ccc|ccc}
 1 & -1 & 3 & 1 & 0 & -3 \\
 0 & 1 & -5 & -2 & 0 & 7 \\
 0 & \textcircled{3} & -16 & -7 & 1 & 21
 \end{array} \right] \\
 \begin{array}{l}
 \text{(-1)} \rightarrow \left[\begin{array}{ccc|ccc}
 1 & -1 & 3 & 1 & 0 & -3 \\
 0 & 1 & -5 & -2 & 0 & 7 \\
 0 & 0 & -1 & -1 & 1 & 0
 \end{array} \right] \\
 \begin{array}{l}
 \text{(+5)} \rightarrow \left[\begin{array}{ccc|ccc}
 1 & -1 & 3 & 1 & 0 & -3 \\
 0 & 1 & -5 & -2 & 0 & 7 \\
 0 & 0 & -1 & -1 & 1 & 0
 \end{array} \right] \\
 \begin{array}{l}
 \text{(+5)} \rightarrow \left[\begin{array}{ccc|ccc}
 1 & 0 & -2 & -1 & 0 & 4 \\
 0 & 1 & \textcircled{-5} & -2 & 0 & 7 \\
 0 & 0 & -1 & -1 & 1 & 0 \\
 0 & 0 & 5 & 5 & -5 & 0
 \end{array} \right] \\
 \begin{array}{l}
 \text{(-2)} \rightarrow \left[\begin{array}{ccc|ccc}
 1 & 0 & -2 & -1 & 0 & 4 \\
 0 & 1 & 0 & 3 & -5 & 7 \\
 0 & 0 & 1 & 1 & -1 & 0 \\
 0 & 0 & 2 & 2 & -2 & 0
 \end{array} \right] \\
 \left[\begin{array}{ccc|ccc}
 1 & 0 & 0 & 1 & -2 & 4 \\
 0 & 1 & 0 & 3 & -5 & 7 \\
 0 & 0 & 1 & 1 & -1 & 0
 \end{array} \right] \Rightarrow A^{-1} = \begin{bmatrix} 1 & -2 & 4 \\ 3 & -5 & 7 \\ 1 & -1 & 0 \end{bmatrix} \\
 \text{3x3 identity matrix} \quad A^{-1}
 \end{array}
 \end{array}
 \end{array}$$

* Note: If, in this process, you get a 0 0 0 row on left side, it means A^{-1} DNE.

Ex 3: Use A^{-1} from Example 2(b) to solve this system of equations.

$$7x - 4y + 6z = 1$$

$$7x - 4y + 5z = 0$$

$$2x - y + z = 7$$

$$A^{-1} = \begin{bmatrix} 1 & -2 & 4 \\ 3 & -5 & 7 \\ 1 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 7 & -4 & 6 & : & 1 \\ 7 & -4 & 5 & : & 0 \\ 2 & -1 & 1 & : & 7 \end{bmatrix}$$

A
(from 2b)

To solve

$$AX=B$$

(where A is an $n \times n$ matrix

X is an $n \times 1$ column vector of variables

B is an $n \times 1$ column vector of constants)

we can left-multiply both sides by A^{-1} . $AX=B$

$$A^{-1}AX = A^{-1}B$$

$$IX = A^{-1}B.$$

$$X = A^{-1}B$$

$AX=B$ (a matrix equ)

$$\begin{bmatrix} 7 & -4 & 6 \\ 7 & -4 & 5 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 7 \end{bmatrix}$$

$$X = A^{-1}B$$

$$X = \begin{bmatrix} 1 & -2 & 4 \\ 3 & -5 & 7 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 7 \end{bmatrix} = \begin{bmatrix} 1(1) + -2(0) + 4(7) \\ 3(1) + -5(0) + 7(7) \\ 1(1) + -1(0) + 0(7) \end{bmatrix}$$

3×3 3×1

$$= \begin{bmatrix} 1+0+28 \\ 3+0+49 \\ 1+0+0 \end{bmatrix} = \begin{bmatrix} 29 \\ 52 \\ 1 \end{bmatrix}$$

another example:

Solve $7x - 4y + 6z = 5$

$$7x - 4y + 5z = 1$$

$$2x - y + z = -1$$

$$\Rightarrow B = \begin{bmatrix} 5 \\ 1 \\ -1 \end{bmatrix}$$

(note: A is same as above $\Rightarrow A^{-1}$ is same also.)

$$X = A^{-1}B$$

$$= \begin{bmatrix} 1 & -2 & 4 \\ 3 & -5 & 7 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 - 2 - 4 \\ 15 - 5 - 7 \\ 5 - 1 + 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix}$$