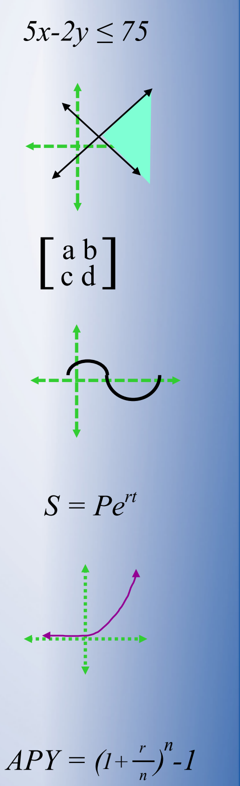

Math 1090 ~ Business Algebra

Section 2.1 Basic Operations with Matrices

Objectives:

- Identify elements of a matrix.
- Differentiate between a scalar and a matrix.
- Identify a square matrix.
- Identify the size of a matrix.
- Determine the transpose of a matrix.
- Write a zero matrix.
- Identify Row or Column Vectors.
- Perform matrix addition and multiplication of a matrix by a scalar.

3x2 matrix ex $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

<u>Vocabulary</u>

matrix: an ordered array of numbers; usually denoted by capital letters

entry ex $a_{11} = 1$ $a_{12} = 2$
$a_{21} = 3$

a_{ij} = the i^{th} row, j^{th} column entry in matrix A

scalar
constant (that we can multiply by a matrix)

order (size)
of a matrix is $m \times n$ (read "m by n") where m = # of rows & n = # of columns

<u>Definitions</u>

A = B the two matrices are same size w/ all the same entries

$B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

A^T (read "A transpose")
exchange rows and columns

$A^T = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$ 2x3 matrix

0 matrix a matrix (of any size) filled w/ zeros
ex $C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Square matrix a matrix that has same # of rows as columns; n x n matrix

ex $P = \begin{bmatrix} 1 & 3 \\ -4 & 2 \end{bmatrix}$

Column or row vector

a matrix that has only one row or one column

ex $D = \begin{bmatrix} 3 \\ 0 \\ 9 \end{bmatrix}$ (column vector) $E = \begin{bmatrix} -1 & 2 & 10 & 15 \end{bmatrix}$ (row vector)

3x1 1x4

Ex 1: For A = $\begin{bmatrix} 3 & 2 & 1 \\ 4 & 0 & -2 \\ 6 & 1 & 5 \end{bmatrix}$

a) size = 3×3

b) a_{13}= 1

(first row, third column entry)

c) $A^T = \begin{bmatrix} 3 & 4 & 6 \\ 2 & 0 & 1 \\ 1 & -2 & 5 \end{bmatrix}$

d) first column vector = $\begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix}$

Ex 2: Given $A = \begin{bmatrix} 1 & 3 & 5 & 7 \\ -5 & 1 & 0 & 1 \\ 3 & -2 & 7 & 0 \end{bmatrix}$

a) What size (order) is A? 3×4

b) What is a_{24}? 1 a_{31}? 3

c) Write a zero matrix the same size as A. $Z = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

d) Find $A^T = \begin{bmatrix} 1 & -5 & 3 \\ 3 & 1 & -2 \\ 5 & 0 & 7 \\ 7 & 1 & 0 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 3 & 5 & 7 \\ -5 & 1 & 0 & 1 \\ 3 & -2 & 7 & 0 \end{bmatrix}$

e) Find -A $= \begin{bmatrix} -1 & -3 & -5 & -7 \\ 5 & -1 & 0 & -1 \\ -3 & 2 & -7 & 0 \end{bmatrix}$

$-A = -1 \cdot A$

Ex 3: Given $A = \begin{bmatrix} 1 & 3 & 1 & 0 \\ 4 & 2 & 1 & 5 \\ -1 & 0 & -2 & 0 \end{bmatrix}$ 3×4 B= $\begin{bmatrix} 2 & 2 & 5 & 1 \\ 0 & 0 & -4 & -3 \\ 1 & 4 & -1 & 2 \end{bmatrix}$ 3×4 C = $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ -1 & 0 & 3 \\ 4 & 5 & 0 \end{bmatrix}$ 4×3

a) Find 2A + B

$$= \begin{bmatrix} 2 & 6 & 2 & 0 \\ 8 & 4 & 2 & 10 \\ -2 & 0 & -4 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 2 & 5 & 1 \\ 0 & 0 & -4 & -3 \\ 1 & 4 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 8 & 7 & 1 \\ 8 & 4 & -2 & 7 \\ -1 & 4 & -5 & 2 \end{bmatrix}$$

b) $A - 3C^T =$

$$A = \begin{bmatrix} 1 & 3 & 1 & 0 \\ 4 & 2 & 1 & 5 \\ -1 & 0 & -2 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ -1 & 0 & 3 \\ 4 & 5 & 0 \end{bmatrix}$$

$$C^T = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 1 & 2 & 0 & 5 \\ 1 & 2 & 3 & 0 \end{bmatrix}$$

$$A - 3C^T = \begin{bmatrix} 1 & 3 & 1 & 0 \\ 4 & 2 & 1 & 5 \\ -1 & 0 & -2 & 0 \end{bmatrix} - \begin{bmatrix} 3 & 6 & -3 & 12 \\ 3 & 6 & 0 & 15 \\ 3 & 6 & 9 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -3 & 4 & -12 \\ 1 & -4 & 1 & -10 \\ -4 & -6 & -11 & 0 \end{bmatrix}$$

① Matrix Addition

A + B =

- we can only add matrices that are the same size!
- add corresponding elements/entries together

② Scalar Multiplication

$cA = [ca_{ij}]$

the matrix we get when we multiply every entry of A by c

Ex 4: Given $A = \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 9 & 1 \end{bmatrix}$ $C = \begin{bmatrix} -3 & 1 & 5 \end{bmatrix}$ $D = \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}$

a) $B^T + D = \begin{bmatrix} 2 \\ 9 \\ 1 \end{bmatrix} + \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \\ 1 \end{bmatrix}$

b) $B - (A-D)^T = \begin{bmatrix} 2 & 9 & 1 \end{bmatrix} - \left(\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} - \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix} \right)^T$

$= \begin{bmatrix} 2 & 9 & 1 \end{bmatrix} - \left(\begin{bmatrix} 6 \\ -2 \\ 3 \end{bmatrix} \right)^T = \begin{bmatrix} 2 & 9 & 1 \end{bmatrix} - \begin{bmatrix} 6 & -2 & 3 \end{bmatrix}$

$= \begin{bmatrix} -4 & 11 & -2 \end{bmatrix}$

c) $(2C + A^T)^T = \left(2\begin{bmatrix} -3 & 1 & 5 \end{bmatrix} + \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}^T \right)^T$

$= \left(\begin{bmatrix} -6 & 2 & 10 \end{bmatrix} + \begin{bmatrix} 4 & 1 & 3 \end{bmatrix} \right)^T$

$= \begin{bmatrix} -2 & 3 & 13 \end{bmatrix}^T$

$= \begin{bmatrix} -2 \\ 3 \\ 13 \end{bmatrix}$