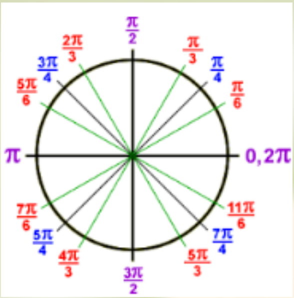
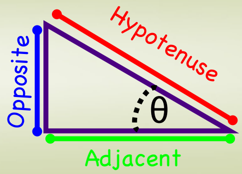
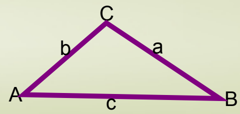
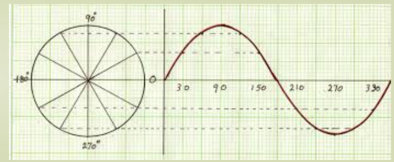








Math 1060 ~ Trigonometry

16 Law of Cosines

$$\sin^2 u + \cos^2 u = 1$$

$$\sin 2u = 2\sin u \cos u$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

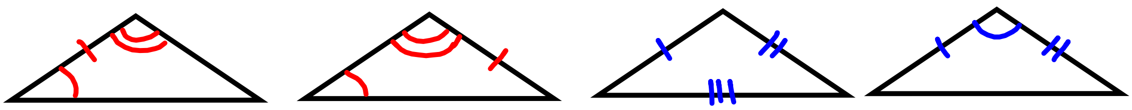
Learning Objectives

In this section you will:

- Use the Law of Cosines to solve oblique triangles.
- Solve SAS and SSS triangles.
- Use Heron's Formula to find the area of a triangle.
- Solve applied problems using the Law of Cosines and the Law of Sines.

Congruence Postulates from Geometry

ASA | AAS | SSS | SAS

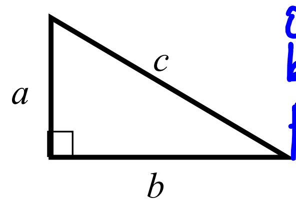


Law of Sines | Law of Sines | Law of Cosines | Law of Cosines

The Law of Cosines is just an adjustment to the Pythagorean Theorem which allows you to apply it to oblique triangles.

In a right triangle with hypotenuse length c,

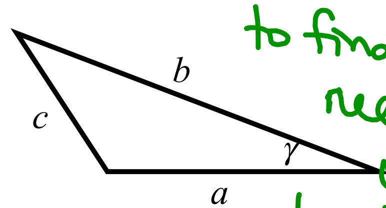
$$c^2 = a^2 + b^2$$



note: if $\gamma = 90°$, then Law of Cosines becomes Pyth. Thm

In any triangle with sides lengths of a,b,c,

$$\boxed{c^2 = a^2 + b^2 - 2ab\cos\gamma}$$

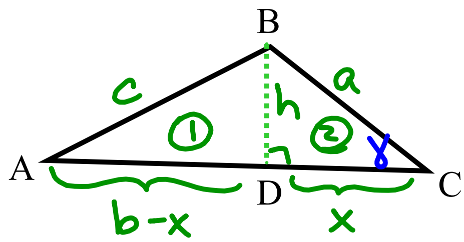


to find c, you need the other 2 leg lengths and the angle included between them

Proof of the Law of Çosines:

Given: $\triangle$ ABC

Prove: $c^2 = a^2 + b^2 - 2ab\cos\gamma$



Draw altitude $\overline{BD}$ to side $\overline{AC}$.

BD = h

① $(b-x)^2+h^2=c^2$ ② $x^2+h^2=a^2$

$h^2=c^2-(b-x)^2$ $h^2=a^2-x^2$

$a^2-x^2=c^2-(b-x)^2$

$c^2=a^2-x^2+(b-x)^2$

$c^2=a^2-\cancel{x^2}+b^2-2bx+\cancel{x^2}$

look in ②:

$\cos\gamma=\frac{x}{a}$

$a\cos\gamma=x$

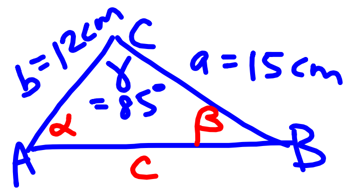
$c^2=a^2+b^2-2bx$

$\boxed{c^2=a^2+b^2-2ab\cos\gamma}$ ✓

As you work these, write the postulate which applies, SSS, SAS, ASA, AAS.

Ex 1: Triangle ABC has a = 15 cm, b = 12 cm, and γ measures 85º. Solve for the missing parts.

SAS case ⇒ use law of Cosines



step1

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

$$c^2 = 15^2 + 12^2 - 2(15)(12)\cos 85°$$

$$c = \sqrt{225 + 144 - 360\cos 85°}$$

$$\boxed{c \simeq 18.37\text{ cm}}$$

step2:

$$\frac{\sin\alpha}{15} = \frac{\sin 85°}{18.37}$$

$$\sin\alpha = \frac{15\sin 85°}{18.37}$$

$$\alpha = \sin^{-1}\left(\frac{15\sin 85°}{18.37}\right) \simeq \boxed{54.41°}$$

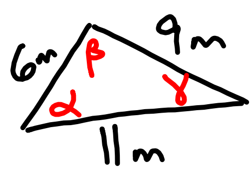
step 3:

$$\beta = 180° - 85° - 54.41°$$

$$\beta \simeq 40.59°$$

Ex 2: Find the angles in a triangle with sides of 6 m, 9 m and 11 m.

SSS case ⇒ Law of Cosines



Step1:

$$11^2 = 9^2 + 6^2 - 2(9)(6)\cos\beta$$

$$121 - 81 - 36 = -108\cos\beta$$

$$\frac{-4}{108} = \cos\beta$$

$$\beta = \cos^{-1}\left(\frac{-4}{108}\right) \simeq \boxed{92.1°}$$

step2:

$$\frac{\sin\gamma}{6} = \frac{\sin 92.1°}{11}$$

$$\sin\gamma = \frac{6\sin 92.1°}{11}$$

$$\gamma = \sin^{-1}\left(\frac{6\sin 92.1°}{11}\right) \simeq \boxed{33.0°}$$

step 3: $\alpha = 180° - 92.1° - 33.0° \simeq \boxed{54.9°}$

There is one more interesting formula for the area of a triangle given the three sides.

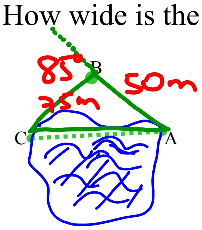
Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}, \qquad s = \dfrac{a+b+c}{2}$

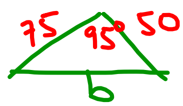
The strategy for solving any triangle, given three parts:

- Draw the triangle.
- Label the parts.
- Determine which law to use. (either Law of Sines or Law of Cosines)
- Solve.

Ex 3: A surveyor is measuring the width of a lake. He stands at point A and walks 50 m to point B, turns counter-clockwise 85º and walks 75 m to point C. How wide is the lake?

dist. from A to C = ?





SAS case

using law of Cosines

$b^2 = 75^2 + 50^2 - 2(50)(75)\cos 95°$

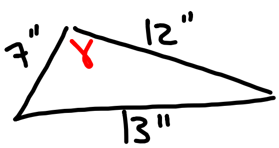
$b^2 \approx 2648.7$

$b \approx \boxed{51.5\text{ m}}$

Ex 4: Find the area of a triangle with sides 7", 12", and 13" in two ways:

a) Use $A = \frac{1}{2}ab\sin\gamma$ th sine in it.

$A = \sqrt{s(s-a)(s-b)(s-c)}, \quad s = \frac{a+b+c}{2}$



$13^2 = 7^2 + 12^2 - 2(7)(12)\cos\gamma$

$169 = 49 + 144 - 168\cos\gamma$

$168\cos\gamma = 24$

$\cos\gamma = \frac{24}{168} = \frac{1}{7}$

$\gamma = \cos^{-1}\left(\frac{1}{7}\right)$

$\boxed{\gamma \approx 81.8°}$

$\rightarrow A = \frac{1}{2}(7)(12)\sin(81.8°)$

$\boxed{A \approx 41.57\text{ in}^2}$

$s = \frac{7+12+13}{2} = 16$

$A = \sqrt{16(16-7)(16-12)(16-13)}$

$= \sqrt{16(9)(4)(3)}$

$= 4 \cdot 3 \cdot 2\sqrt{3}$

$= 24\sqrt{3}$

$\approx \boxed{41.57\text{ in}^2}$

Trigonometry and Bearings

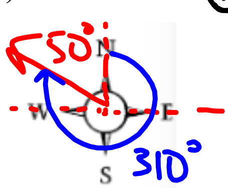
In surveying and navigation, directions are often given in terms of bearings. This can be in one of two ways.

1 a) Expresses as some east or west angle from north or south.

2 b) Expressed as degrees in a clockwise direction from north.

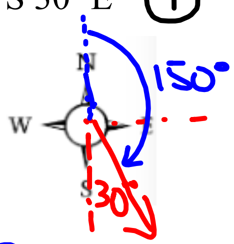
Ex 5: Sketch each bearing and express it in both ways.

a) N 50º W 1



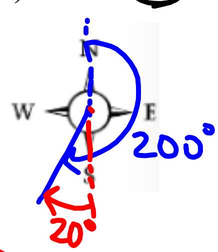
2 310°

b) S 30º E 1



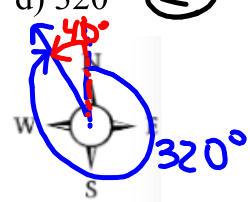
2 150°

c) 200º 2



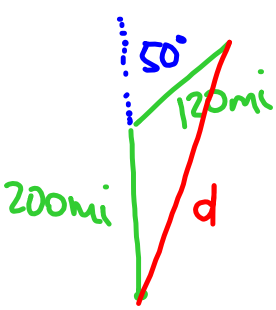
1 S20°W

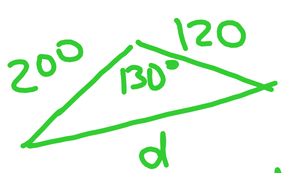
d) 320º 2



1 N40°W

Ex 6: A plane flies due north for 200 miles, then turns to a bearing of 50º and flies 120 miles. How far is it from the starting point?





Use Law of Cosines:

$$d^2 = 200^2 + 120^2 - 2(120)(200)\cos 130^\circ$$

$$d^2 \simeq 72030$$

$$d \simeq \boxed{268.4 \text{ mi}}$$