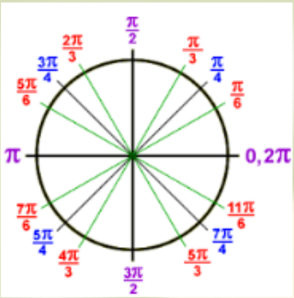
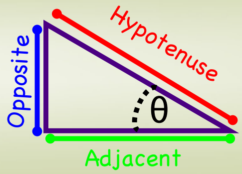
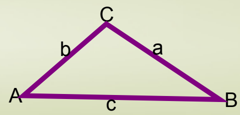
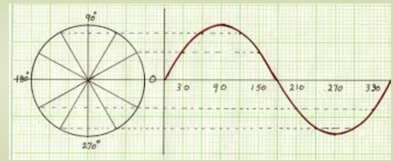








$\sin^2 u + \cos^2 u = 1$

$\sin 2u = 2\sin u \cos u$

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$c^2 = a^2 + b^2 - 2ab\cos C$

Math 1060 ~ Trigonometry

13 Solving Trigonometric Equations

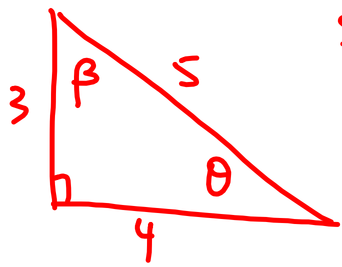
Learning Objectives

In this section you will:

- **Use inverse trigonometric functions to solve right triangles.**
- **Use inverse trigonometric functions to solve for angles in trigonometric equations.**
- **Write complete real solutions to equations containing a single trigonometric function.**
- **Evaluate exact solutions in the interval $[0,2\pi)$.**
- **Use inverse trigonometric functions to solve real-world applications.**

The inverse functions allow us to calculate angles in a right triangle, given two of the sides.

Ex 1: Determine the acute angles in a 3-4-5 right triangle.



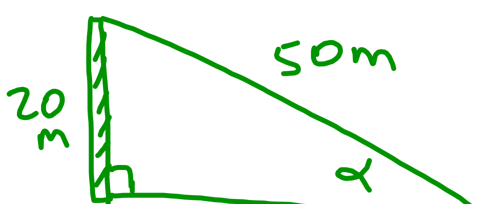
$\sin\theta = \frac{3}{5}$ OR

$\boxed{\theta = \sin^{-1}\left(\frac{3}{5}\right)}$ $\theta = \cos^{-1}\left(\frac{4}{5}\right)$

$\approx 36.87°$

$\tan\beta = \frac{4}{3} \Rightarrow \beta = \tan^{-1}\left(\frac{4}{3}\right) \approx 53.13°$

Ex 2: If a 50-meter rope is attached to the top of a 20-meter pole for a tight-rope event, what angle does the rope make with the ground?



$\sin\alpha = \frac{20\,\cancel{m}}{50\,\cancel{m}}$

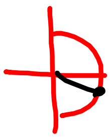
$\alpha = \sin^{-1}\left(\frac{2}{5}\right) \approx 23.58°$

We can also solve trigonometric equations for angles in radians.

Remember: $x = \sin^{-1}(a)$ returns a single, principal value and $\sin x = a$ will have an infinite number of solutions, if defined.

Sample: Solve for x.

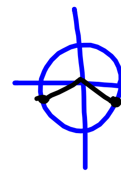
① $x = \sin^{-1}\left(-\frac{1}{2}\right)$ (has only one answer)



$x = -\frac{\pi}{6}$

② $\sin x = -\frac{1}{2}$ (has ∞ solutions)

$$x = \begin{cases} -\frac{\pi}{6} + 2n\pi \\ \frac{7\pi}{6} + 2n\pi \end{cases}$$



$n \in \mathbb{Z}$ (n is an integer)

Ex 3: Solve these for x, where x is in radians. State the solution on the ① interval $[0,2\pi)$ and then state the ② general solution for all angles which provide a solution to the equation.

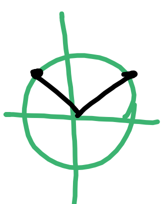
Note: $\sec^2 x = (\sec x)^2$

a) $\sqrt{2}\sin x - 1 = 0$

$\sqrt{2}\sin x = 1$

$\sin x = \frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$

$\sin x = \frac{\sqrt{2}}{2}$



① $x = \frac{\pi}{4}, \frac{3\pi}{4}$

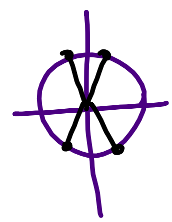
$x \in [0, 2\pi)$

② $x = \begin{cases} \frac{\pi}{4} + 2n\pi \\ \frac{3\pi}{4} + 2n\pi \end{cases}$ $n \in \mathbb{Z}$

b) $\sec^2 x = 4$

$\frac{1}{\cos^2 x} = 4$

$\cos^2 x = \frac{1}{4}$



$\cos x = \pm\frac{1}{2}$

$x = \pm\frac{\pi}{3}, \pm\frac{2\pi}{3}$

① $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$x \in [0, 2\pi)$

② $x = \begin{cases} \pm\frac{\pi}{3} + 2n\pi & \text{(in Q1+4)} \\ \pm\frac{2\pi}{3} + 2n\pi & \text{(in Q2+3)} \end{cases}$

OR

$x = \pm\frac{\pi}{3} + n\pi$

$n \in \mathbb{Z}$

Ex 4: State the general solution for each of these.

a) $\tan^2 x - 3 = 1$

$\tan^2 x = 4$

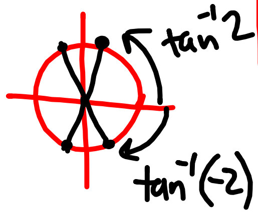
$\tan x = \pm 2$

$x = \tan^{-1} 2$

$\quad \tan^{-1}(-2)$

$\quad \tan^{-1}(-2) + \pi$ (in Q2)

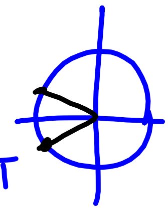
$\quad \tan^{-1}(2) + \pi$ (in Q3)



$$\boxed{x = \begin{cases} \tan^{-1}(2) + n\pi \\ \tan^{-1}(-2) + n\pi \end{cases} \quad n \in \mathbb{Z}}$$

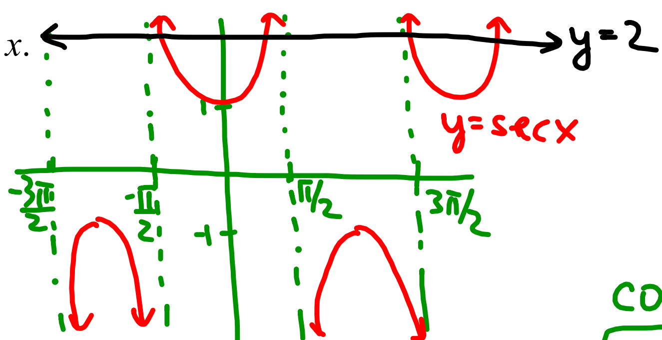
b) $\cos(2x) = -\dfrac{\sqrt{3}}{2}$

$$2x = \begin{cases} \dfrac{5\pi}{6} + 2n\pi \\ \dfrac{7\pi}{6} + 2n\pi \end{cases} \quad n \in \mathbb{Z}$$



$$\boxed{x = \begin{cases} \dfrac{5\pi}{12} + n\pi \\ \dfrac{7\pi}{12} + n\pi \end{cases}}$$

Ex 5: State all radian values where the line $y = 2$ intersects with the function $y = \sec x$.



when is $2 = \sec x$?

$\cos x = \frac{1}{2}$

$$\boxed{x = \pm\frac{\pi}{3} + 2n\pi}$$

$n \in \mathbb{Z}$