3.7 ~ Graphing polar equations

In this lesson you will:

- Graph polar equations by point plotting.
- Use symmetry, zeros and maximum *r*-values to sketch graphs of polar equations.
- Recognize special polar graphs.

What do these equations represent?

$\theta = \beta$

$r\cos\theta = a$

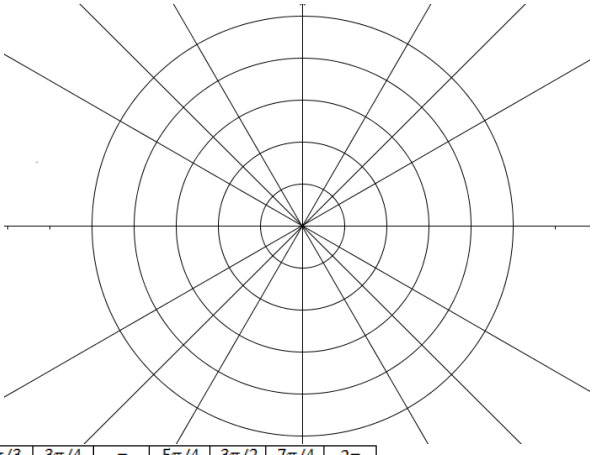
$r\sin\theta = b$

What about these?

$r = 2a\cos\theta$

$r = 2b\sin\theta$

$r = 2a\cos\theta + 2b\sin\theta$

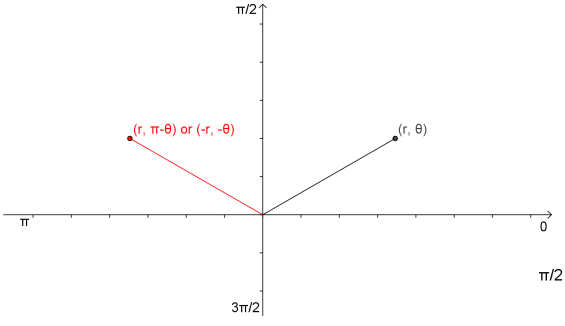


Example 1:

$r = 4\cos\theta$

θ	0	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
r											

Symmetry

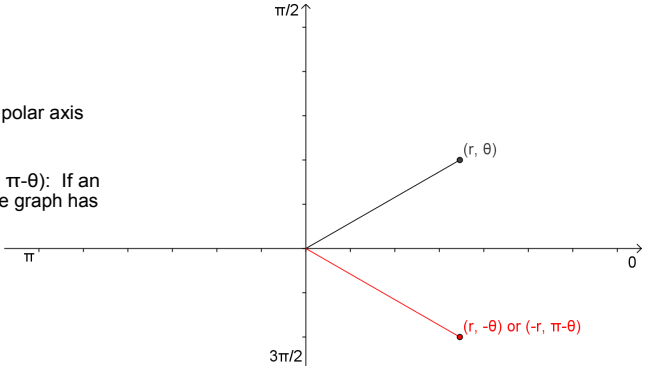


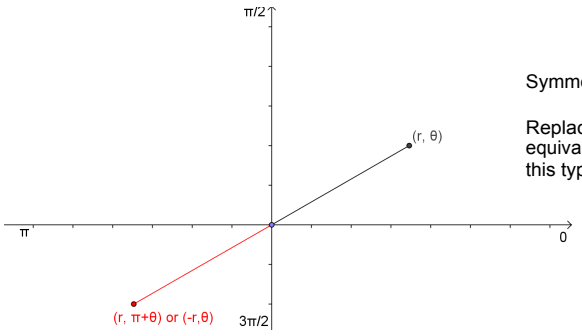
Symmetry with respect to the line θ=π/2

Replace (r,θ) with (r,π-θ) or (-r, -θ): If an equivalent equation results, the graph has this type of symmetry.

Symmetry with respect to the polar axis (θ=0):

Replace (r,θ) with (r,-θ) or (-r, π-θ): If an equivalent equation results, the graph has this type of symmetry.





Symmetry with respect to the pole

Replace (r,θ) with (-r,θ) or (r, π+θ): If an equivalent equation results, the graph has this type of symmetry.

If a polar equation passes a symmetry test, then its graph definitely exhibits that symmetry. However, if a polar equation fails a symmetry test, then its graph may or may not have that kind of symmetry.

Zeros and maximum *r*-values

Other helpful tools in graphing polar equations are knowing the values for θ for which |*r* | is maximum and those for which *r* = *0*.

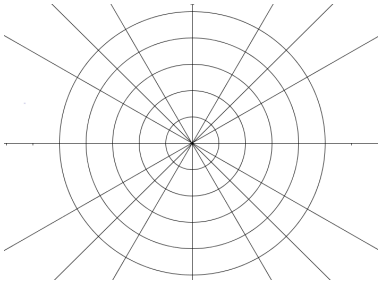
Example 2 : Graph $r = \frac{1}{2} + \cos\theta$

Symmetry:

|*r* | maximum:

Zero of *r* :

θ	0	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	π
r							

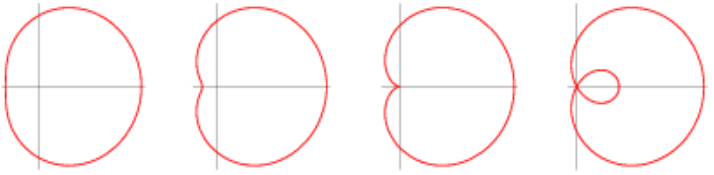


Limaçons

$$r = a \pm b\cos\theta$$

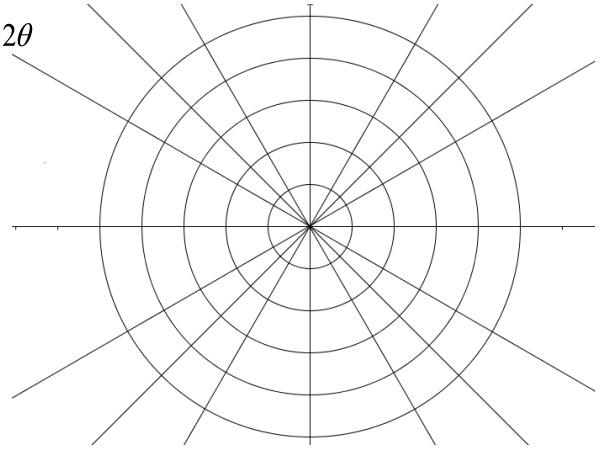
$$r = a \pm b\sin\theta$$

$$a > 0,\ \ b > 0$$



$\frac{a}{b} \geq 2$	$1 < \frac{a}{b} < 2$	$\frac{a}{b} = 1$	$\frac{a}{b} < 1$
Convex limaçon	Dimpled limaçon	Cardioid -always passes through pole	Limaçon with inner loop

Example 3: Graph $r = 3\sin 2\theta$



Symmetry:

|*r*| maximum:

Zero of *r*:

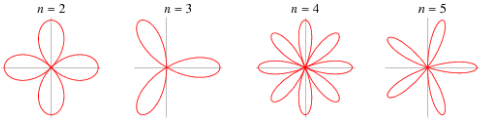
θ	0	π/8	π/6	π/4	π/3	π/2
r						

Roses

$$r = a \sin(n\theta),$$

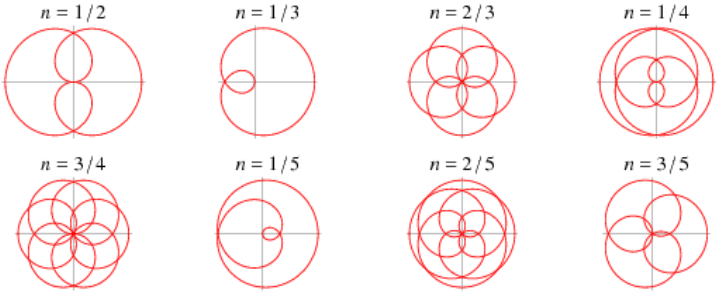
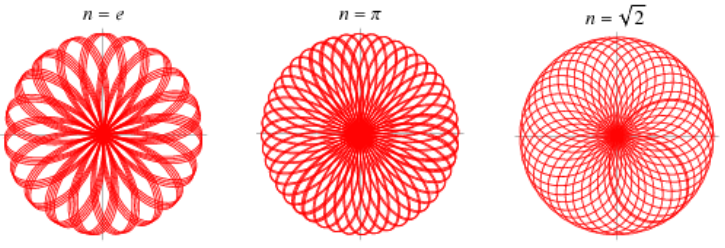
or

$$r = a \cos(n\theta).$$



If n is odd, the rose is n-petalled. If n is even, the rose is $2n$-petalled.

No reason to limit ourselves to n integer:



Or even rational: