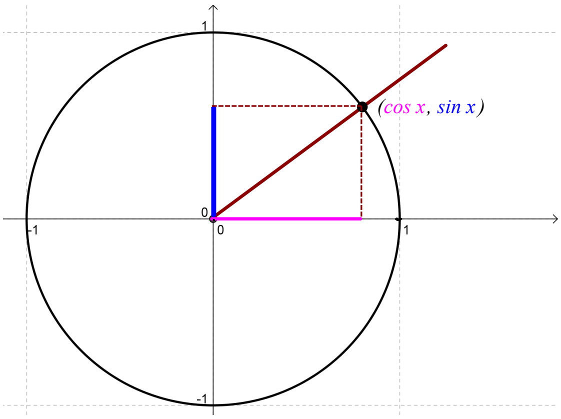
1.5 ~ Graphs of Sine and Cosine Functions

In this lesson you will:

- Sketch the graphs of basic sine and cosine functions.
- Use amplitude and period to help sketch graphs.
- Sketch translations of these functions.

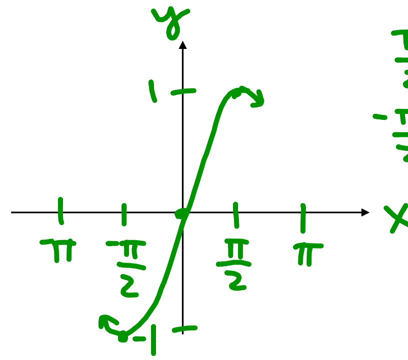
$f(x) = \sin x$

http://tube.geogebra.org/student/m45354?mobile=true



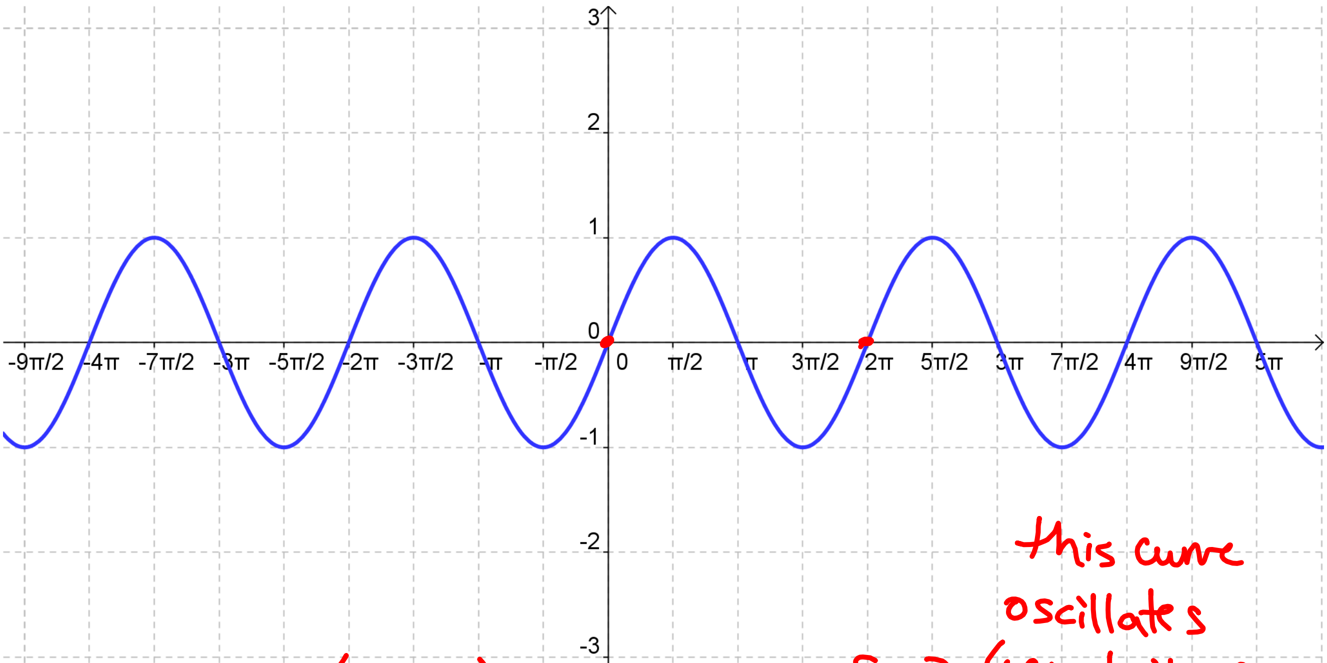
$-1 \le \sin x \le 1$

$y = \sin x$



x	$y = \sin x$
0	0
$\frac{\pi}{2}$	1
$-\frac{\pi}{2}$	-1

Graph of $f(x) = \sin x$



Domain: $(-\infty, \infty)$

Range: $[-1, 1]$

this curve oscillates (repeats itself in shape)

Period: 2π

(horizontal length until shape repeats itself)

Symmetry:

wrt origin. (odd fn)

$f(x) = \cos x$

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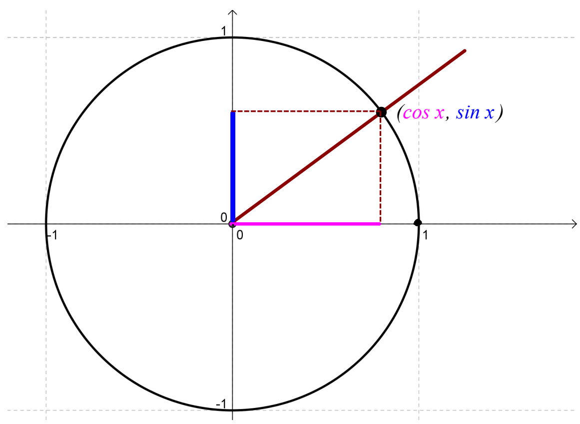
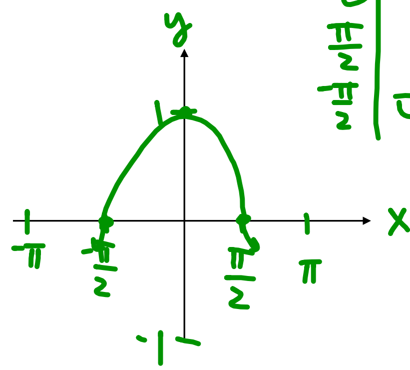
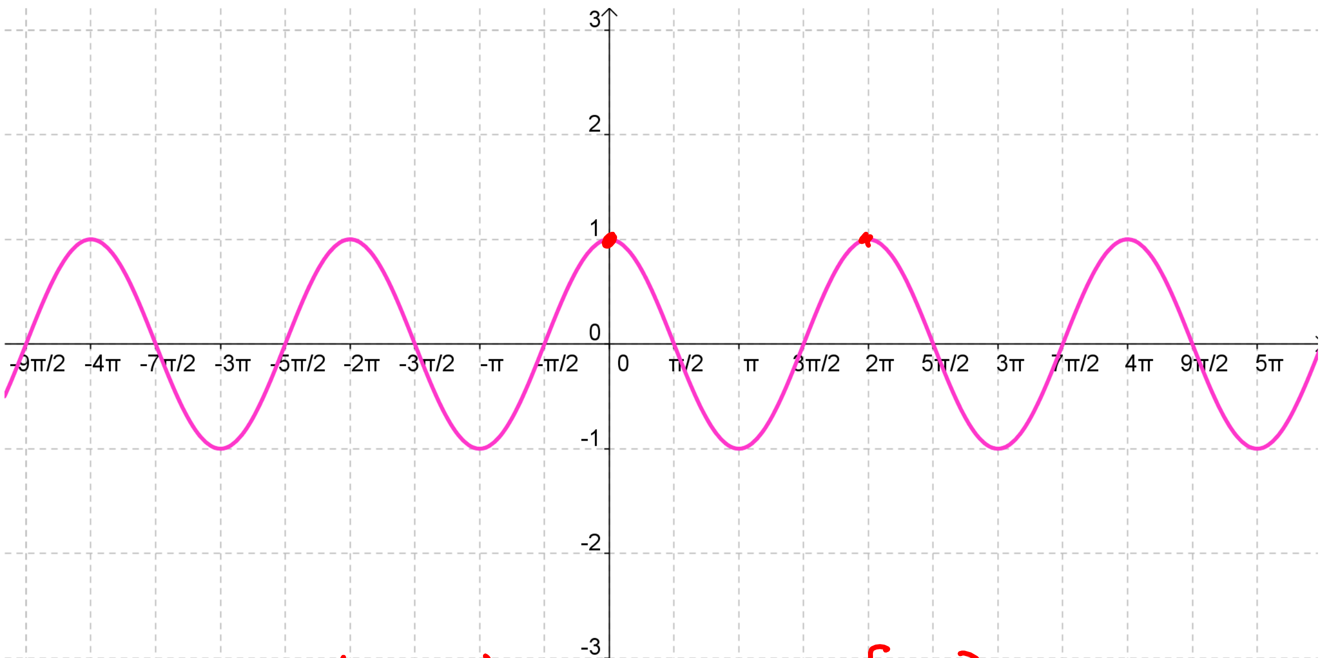




x	$\cos x$
0	1
$\frac{\pi}{2}$	0
$-\frac{\pi}{2}$	0

Graph of $f(x) = \cos x$



Domain: $(-\infty, \infty)$

Range: $[-1, 1]$

Period: 2π

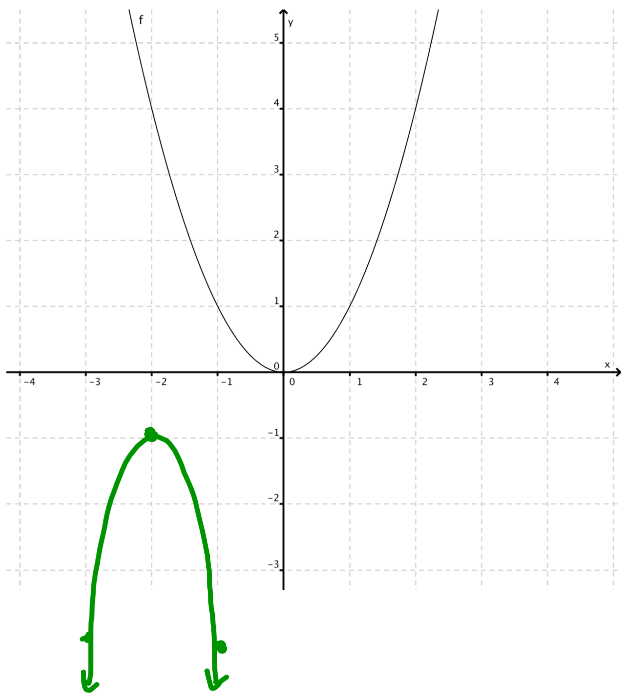
Symmetry: wrt y-axis (even fn)

How can you graph $y = 2\sin(x - \frac{\pi}{3}) + 1$?

This is a transformation of the basic $y = \sin x$ curve.

It may help to remember transformations to one of the algebraic functions.

How does the graph of $y = -3(x+2)^2 - 1$ relate to the graph of $y = x^2$?



vert. reflection

$y = -3(x+2)^2 - 1$

vert. stretch by factor of 3

horiz. shift left 2

vert. shift down 1

In general, remember the effect of a, h and k on the graph of $y = x^2$.

$y = a(x-h)^2 + k$

(h,k) new vertex $\left(\begin{array}{l} h = \text{horiz. shift} \\ k = \text{vert. shift} \end{array} \right)$

$|a|$ = vert. "stretch" factor $\left(\begin{array}{l} \text{if } |a|>1, \text{ stretch} \\ \text{if } |a|<1, \text{ shrink} \end{array} \right)$

$\left\{ \begin{array}{l} \text{if } a>0, \text{ no vert. reflection (concave up)} \\ \text{if } a<0, \text{ vert. reflection (concave down)} \end{array} \right.$

$y = a\ sin(bx+c)+d$

What effect do *a, b, c* and *d* have on the graph of trigonometric functions?

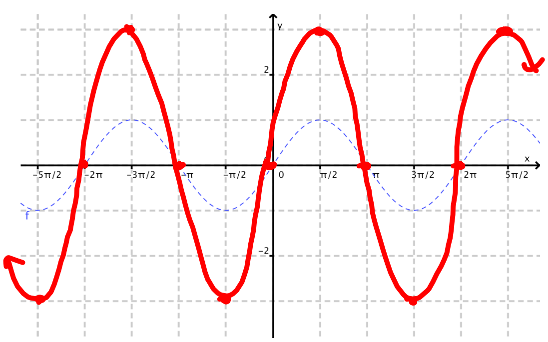
Let's look at it one part at a time:

$y = a\ sin\ x$ $|a|$ = vertical "stretch" factor

Amplitude: |a| amplitude = $\frac{1}{2}$ the distance from lowest to highest points on graph.

Example 1: Graph each of these.

$y = 3sin\ x$ amp. = 3



$y = -2cos\ x$ amp. = 2

reflected vertically

$y = sin(bx)$

creates a horizontal stretch (if $|b| < 1$)

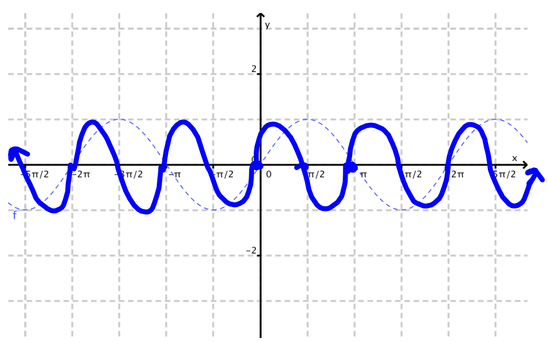
or shrink (if $|b| > 1$)

$\boxed{\text{Period} = \frac{2\pi}{|b|}}$

Example 2: Graph each of these.

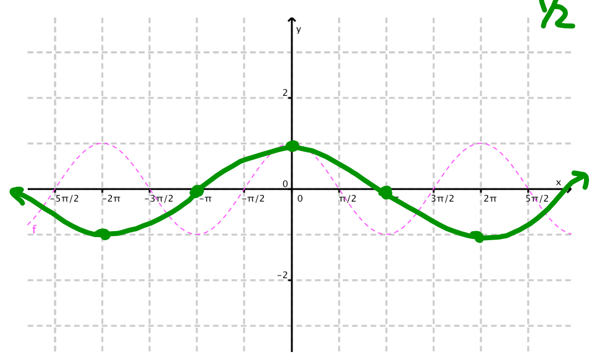
(horizontal shrink)

$y = sin(2x)$ period $= \frac{2\pi}{2} = \pi$



(horizontal stretch)

$y = cos(\frac{1}{2}x)$ Period $= \frac{2\pi}{1/2} = 4\pi$



$y = sin(x - c)$

Horizontal shift = $c > 0$, shift right by c

$c < 0$, shift left by c.

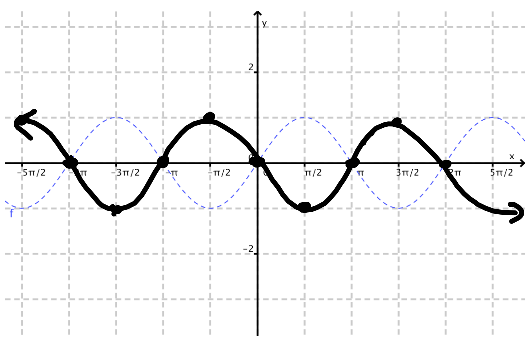
Example 3: Graph each of these.

$c = -\pi \Rightarrow$ shift left π

$y = sin(x+\pi) = \sin(x-(-\pi))$

$y = cos(x - \frac{\pi}{2})$ $c = \frac{\pi}{2}$

horiz. shift right by $\frac{\pi}{2}$



(note: $\cos(x-\frac{\pi}{2}) = \sin x$)

$y = sin(bx - c)$ $= \sin\left(b\left(x - \frac{c}{b}\right)\right)$

Period = $\frac{2\pi}{b}$

Horizontal shift = $\frac{c}{b}$ $\left(\begin{array}{ll} \frac{c}{b} > 0 & \text{to the right} \\ \frac{c}{b} < 0 & \text{" " left} \end{array}\right)$

Example 4: Graph each of these.

1 period $= \frac{2\pi}{2} = \pi$

2 horiz. shift $\frac{\pi}{2}$ to right

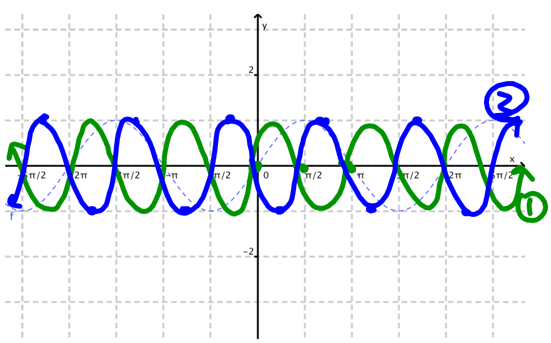
$y = sin(2x - \pi)$ $= \sin\left(2\left(x - \frac{\pi}{2}\right)\right)$

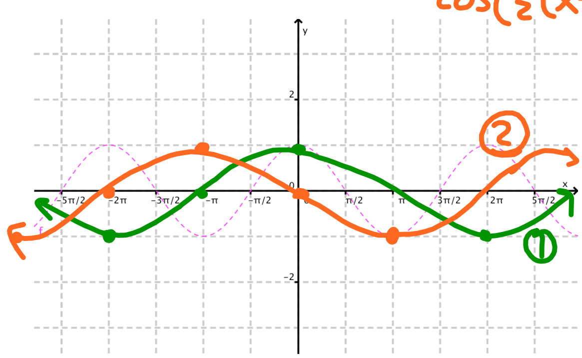
1 period $= \frac{2\pi}{1/2} = 4\pi$

horiz. shift π to left

2 $y = cos((\frac{1}{2})x + \frac{\pi}{2})$ $= \cos\left(\frac{1}{2}(x+\pi)\right)$

$= \cos\left(\frac{1}{2}(x-(-\pi))\right)$





$y = \sin(bx - c)$ $= \sin\left(b\left(x - \frac{c}{b}\right)\right)$

Period = $\frac{2\pi}{b}$

Horizontal shift = $\frac{c}{b}$ $\left(\begin{array}{ll} \frac{c}{b} > 0 & \text{to the right} \\ \frac{c}{b} < 0 & \text{" " left} \end{array}\right)$

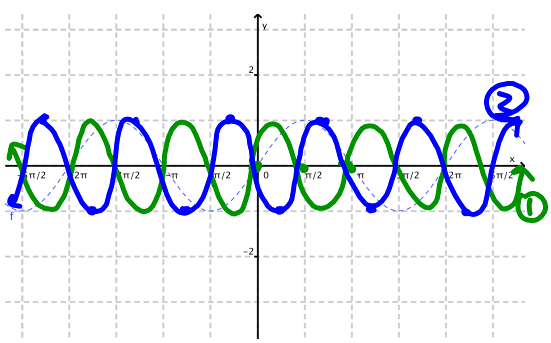
Example 4: Graph each of these.

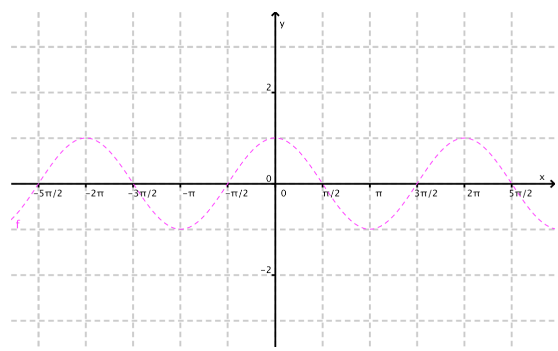
① period = $\frac{2\pi}{2} = \pi$

② horiz. shift $\frac{\pi}{2}$ to right

$y = \sin(2x - \pi) = \sin\left(2\left(x - \frac{\pi}{2}\right)\right)$

$y = \cos\left(\left(\frac{1}{2}\right)x + \frac{\pi}{2}\right)$





$y = sin(x) + d = sin\ x + d$

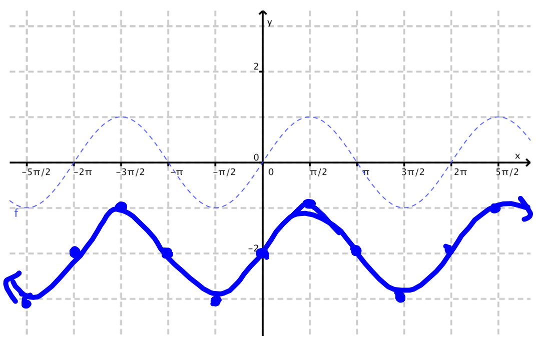
Vertical Shift : $d > 0$ shift up d

$d < 0$ shift down d

Example 5: Graph each of these.

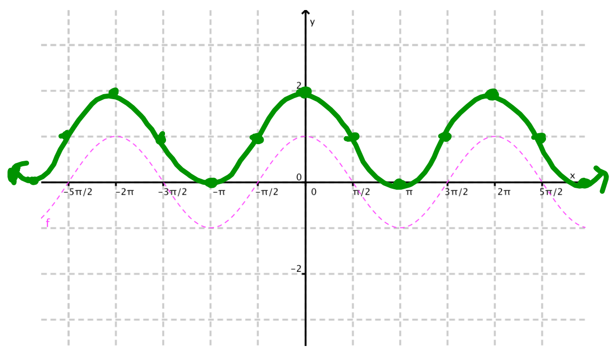
shift down 2

$y = sin\ x - 2$ = $\sin x + -2$



shift up 1

$y = cos\ x + 1$

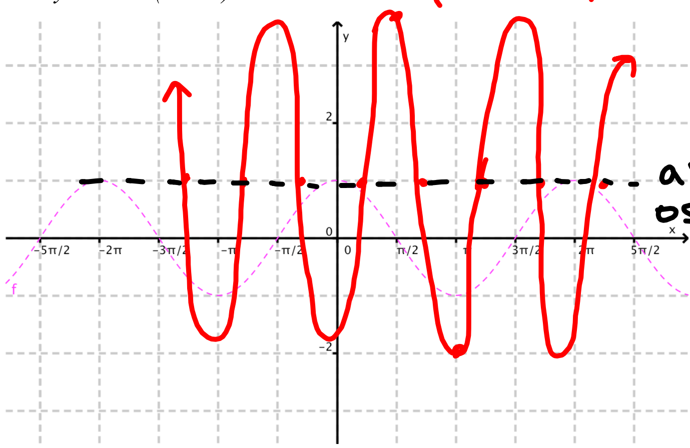


So, when we graph a sine or cosine function there are these things to consider:

Amplitude
Period
Phase shift (horizontal)
Vertical shift

Example 6: Sketch this function.

$y = 3\cos(2x - \pi) + 1 = 3\cos\left(2\left(x - \frac{\pi}{2}\right)\right) + 1$



axis of oscillation

Amplitude 3

Period $\frac{2\pi}{2} = \pi$

Phase shift (horizontal) $\frac{\pi}{2}$ (right)

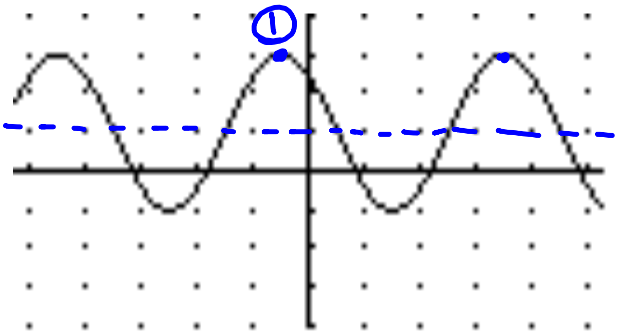
Vertical shift 1 (up)

$y = \cos x$	$y = \cos(2x)$	$y = 3\cos(2x)$	$y = 3\cos\left(2\left(x - \frac{\pi}{2}\right)\right) + 1$
$(0, 1)$	$(0, 1)$	$(0, 3)$	$\left(\frac{\pi}{2}, 4\right)$
$\left(\frac{\pi}{2}, 0\right)$	$\left(\frac{\pi}{4}, 0\right)$	$\left(\frac{\pi}{4}, 0\right)$	$\left(\frac{3\pi}{4}, 1\right)$
$(\pi, -1)$	$\left(\frac{\pi}{2}, -1\right)$	$\left(\frac{\pi}{2}, -3\right)$	$(\pi, -2)$
$\left(\frac{3\pi}{2}, 0\right)$	$\left(\frac{3\pi}{4}, 0\right)$	$\left(\frac{3\pi}{4}, 0\right)$	$\left(\frac{5\pi}{4}, 1\right)$

Example 7: Look at each of these graphs and write an equation in the form of

$y = a \sin(b(x-h))+k$ or $y = a \cos(b(x-h)) + k$

x-axis tic marks = $\frac{\pi}{2}$, y-axis tic marks = 1



$y=1$

axis of oscillation

(cosine graph)

amp = 2

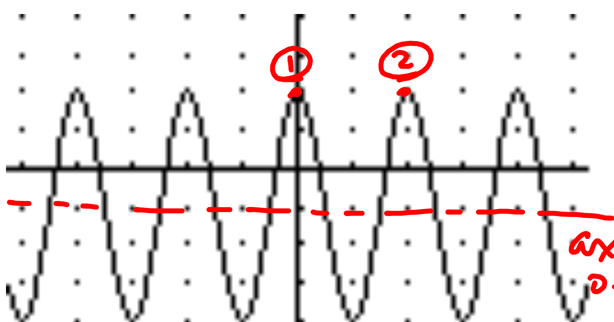
period $= 4\left(\frac{\pi}{2}\right) = 2\pi$

(no horiz stretch)

shift up 1

shift left $\frac{\pi}{4}$

$$\boxed{y = 2\cos\left(x + \frac{\pi}{4}\right) + 1}$$



axis of oscillation

$$\boxed{y = 3\cos(2x) - 1}$$

choose cosine curve

amp = 3

shift down 1

no horiz. shift

$2\left(\frac{\pi}{2}\right) = \pi = \text{period}$

$\frac{2\pi}{b} = \pi \Rightarrow b = 2$

Here are some applets in case you want to play with the transformation variables.

http://www.analyzemath.com/trigonometry/sine.htm

http://tube.geogebra.org/student/m45354?mobile=true