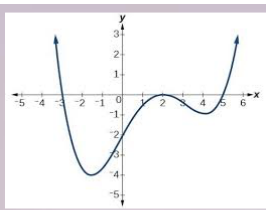
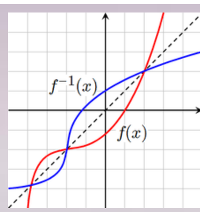
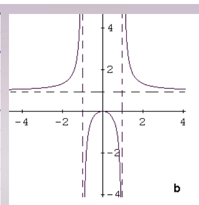
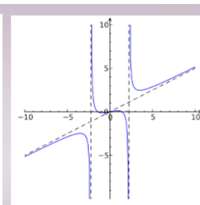
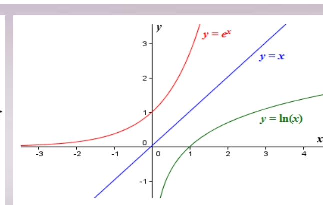










Math 1050 ~ College Algebra

6 Quadratic Functions

$$-3x + 4y = 5$$
$$2x - y = -10$$
$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^{m} k = \frac{m(m+1)}{2}$$
$$\sum_{k=0}^{n} z^k = \frac{1 - z^{n+1}}{1 - z}$$

Learning Objectives

- Graph a quadratic function through transformations of $f(x) = x^2$.
- Change a quadratic function from general to standard form.
- Find the vertex and axis of symmetry of a quadratic function.
- Find the intercepts of a quadratic function.
- Graph a quadratic function using vertex, axis of symmetry and intercepts.
- Solve applications that require finding the maximum or minimum value of a quadratic function.

Quadratic Functions

A polynomial function: $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1 x + a_0$.

a_0 = constant

n = degree of polynomial (a whole number)

a_n = leading coefficient

A quadratic function is a type of polynomial function where the degree = 2.

$\boxed{f(x) = ax^2 + bx + c}$ $\quad a,b,c \in \mathbb{R},\ a \neq 0$

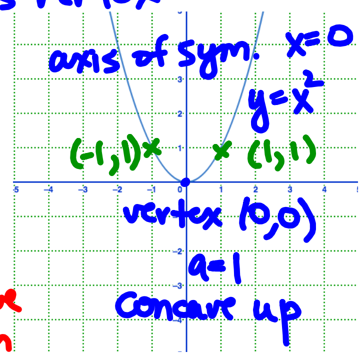
constants

general form	standard form aka (vertex form)
$f(x) = ax^2 + bx + c$	$f(x) = a(x - h)^2 + k$
vertex $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$	(h,k) is vertex

axis of symmetry: vertical line, w/ eqn $x=h$ (graphically behaves like a mirror)

vertex: turning pt of parabola (h,k)

concavity: up or down; ① if $a>0$, concave up; ② if $a<0$, concave down



Ex 1: Determine the vertex, axis of symmetry and concavity of each of these.

a) $f(x) = 3x^2 + 6x - 4$ $\quad a=3,\ b=6,\ c=-4$

(general form)

vertex: $\frac{-b}{2a} = \frac{-6}{2(3)} = -1$

$\boxed{(-1,-7)}$

$f(-1) = 3(-1)^2 + 6(-1) - 4$
$= 3 - 6 - 4 = -7$

axis of symmetry: $\boxed{x=-1}$

Concavity: $a = 3 > 0$
$\Rightarrow$ parabola is $\boxed{\text{concave up}}$

b) $f(x) = -2(x+3)^2 - 4$ $\quad a=-2,\ h=-3,\ k=-4$

(standard form)

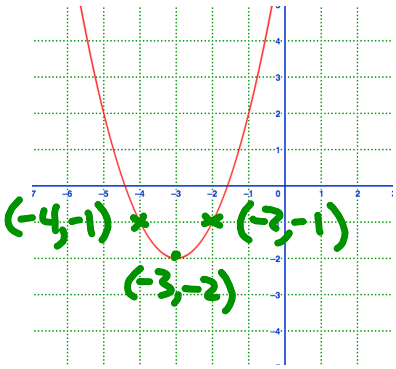
vertex: $\boxed{(-3,-4)}$

axis of symmetry: $\boxed{x=-3}$

concavity: $a=-2<0$
$\Rightarrow$ parabola is $\boxed{\text{concave down}}$

Ex 2: Write the equation of this quadratic function in standard form, then use algebra to write it in general form.

standard form: $y = a(x-h)^2 + k$



vertex: $(-3,-2)$

concave up $\Rightarrow a > 0$

$a = 1$

$\Rightarrow$ our parabola is

$y = 1(x-(-3))^2 + (-2) = (x+3)^2 - 2$

$y = (x+3)(x+3) - 2 = (x^2 + 3x + 3x + 9) - 2$

$\boxed{y = x^2 + 6x + 7}$ general form

Ex 3: Put this equation in standard form and sketch a graph of it.

$$y = \underline{-2x^2 + 4x} + 2$$

(we must complete the square)

$y = -2(x^2 - 2x) + 2$

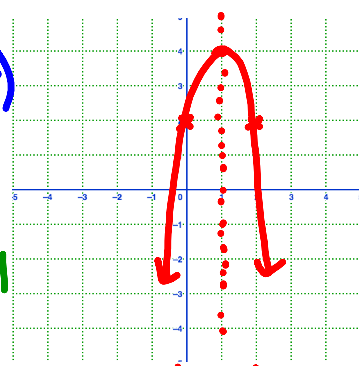
$= -2(x^2 - 2x + 1)$

$+2 - 2(-1)$

$= -2(x-1)^2 + 2 + 2$

$y = -2(x-1)^2 + 4$

$\left(\frac{-2}{2}\right)^2 = 1$



vertex: $(1,4)$

$a = -2$, concave down

axis of symmetry $x = 1$

Finding Roots of Quadratic Functions

roots and zeros of a polynomial fn are synonymous

To find the roots, solve for $f(x) = 0$.

x-values

If the expression on the left factors, set each factor equal to 0 and solve for *x*.

If you prefer not to factor, or it does not factor, you can always use the Quadratic Formula.

Quadratic Formula $\boxed{ax^2 + bx + c = 0 \Rightarrow x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$

Ex 4: Determine the roots of each of these.

a) $f(x) = 3x^2 + 5x - 4$

$3x^2+5x-4=0$

use quadratic formula

$a=3,\ b=5,\ c=-4$

$x=\dfrac{-5\pm\sqrt{25-4(3)(-4)}}{2(3)}$

$x=\dfrac{-5\pm\sqrt{25+48}}{6}$

$\boxed{x=\dfrac{-5\pm\sqrt{73}}{6}} \simeq -2.26$ or 0.59

(two x-values)

b) $f(x) = 9x^2 - 6x + 1$

$9x^2-6x+1=0$

$(3x-1)(3x-1)=0$

$(3x-1)=0$

$3x=1$

$x=\frac{1}{3}$

c) $f(x) = 4x^2 - 6x - 3$

$4x^2-6x-3=0$

use quadratic formula

$a=4,\ b=-6,\ c=-3$

$x=\dfrac{6\pm\sqrt{36-4(4)(-3)}}{2(4)}$

$x=\dfrac{6\pm\sqrt{36+48}}{8}$

$x=\dfrac{6\pm\sqrt{84}}{8}$

$x=\dfrac{6\pm2\sqrt{21}}{8}=\dfrac{2(3\pm\sqrt{21})}{8}$

$\boxed{x=\dfrac{3\pm\sqrt{21}}{4}} \simeq 1.9$ or -0.40

In the quadratic formula, the expression inside the radical is called the <u>discriminant</u>. It determines whether there is one real root, two real roots or no real roots.

$b^2 - 4ac$ = discriminant (it discriminates the roots)

① $b^2-4ac>0$ 2 roots ② $b^2-4ac=0$ one root ③ $b^2-4ac<0$ no roots

Ex 5: Find the discriminant of the equations in example 4.

a) $f(x) = 3x^2 + 5x - 4$

$b^2-4ac = 25-4(3)(-4)$
$= 25+48 > 0$
$\Rightarrow \boxed{\text{2 real roots}}$

b) $f(x) = 9x^2 - 6x + 1$

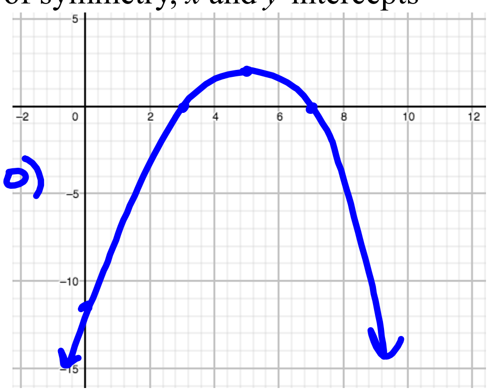
$b^2-4ac = 36-4(9)(1)$
$= 36-36 = 0$
$\Rightarrow \boxed{\text{one real root}}$

c) $f(x) = 4x^2 - 6x - 3$

$b^2-4ac = 36-4(4)(-3)$
$= 36+48 = 84 > 0$
$\Rightarrow \boxed{\text{2 real roots}}$

Ex 6: For this function, find the vertex, axis of symmetry, x and y-intercepts and sketch it.

$$f(x) = -\frac{1}{2}(x^2 - 10x + 21)$$



<u>x-intercepts:</u> $(3,0)$ and $(7,0)$

$0 = -\frac{1}{2}(x^2-10x+21)$
$0 = -\frac{1}{2}(x-3)(x-7)$
$x-3=0$ or $x-7=0$
$x=3$ $x=7$

<u>y-intercept:</u> $(0, -\frac{21}{2})$

$y = -\frac{1}{2}(0^2-10(0)+21)$
$= -\frac{1}{2}(21) = -\frac{21}{2} = -10.5$

<u>vertex:</u> $(5,2)$

<u>axis of sym.:</u> $x=5$

vertex x-value is halfway between (i.e. the average) the roots of $f(x)$.

$x = \frac{3+7}{2} = 5$

$f(5) = -\frac{1}{2}(5^2-10(5)+21)$
$= -\frac{1}{2}(-4) = 2$

An Application Problem

Ex 7: The height of an object shot straight up in the air from a height of 128 feet at an initial velocity of 32 ft/sec is modeled by $h(t) = -16t^2 + 32t + 128$, where t = time.

Determine the ① maximum height the object reaches and ② the time it will hit the ground.

find the vertex:

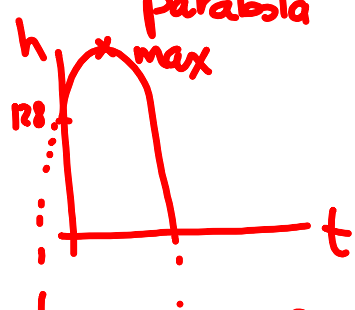
$a = -16,\ b = 32,\ c = 128$

$t = \frac{-32}{2(-16)} = \boxed{1 \text{ sec}}$

$h(1) = -16(1^2) + 32(1) + 128$

$= 16 + 128 = \boxed{144 \text{ ft}}$ ①

graph of $h(t)$ is concave down parabola



② hits the ground when $h = 0$

$0 = -16t^2 + 32t + 128$

$0 = -16(t^2 - 2t - 8)$

$0 = t^2 - 2t - 8$

$0 = (t - 4)(t + 2)$

$t - 4 = 0$ or $t + 2 = 0$

$t = 4$ or $\cancel{-2}$

$\boxed{t = 4 \text{ sec}}$