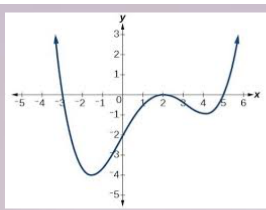
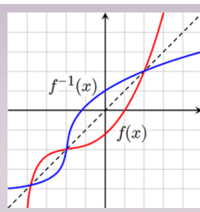
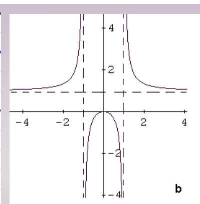
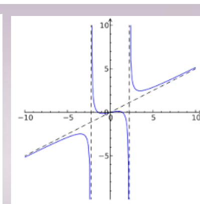
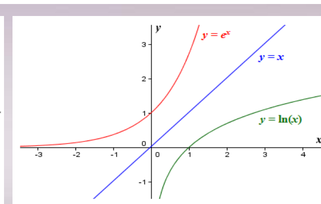










Math 1050 ~ College Algebra

20 Applications of Exponentials and Logarithms

$$-3x + 4y = 5$$
$$2x - y = -10$$
$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^{m} k = \frac{m(m+1)}{2}$$
$$\sum_{k=0}^{n} z^k = \frac{1 - z^{n+1}}{1 - z}$$

Learning Objectives

- Use compound interest formulas to solve financial applications.
- Solve applications of uninhibited growth and decay.
- Solve additional applications represented by exponential and logarithmic models.

Simple, Compound and Continuously Compounded Interest

Ex 1: If you invest \$100 at a yearly interest rate of 5%, show how it will grow during the first five years.

$r=0.05$

	Simple Interest	Compound Interest
$t=1$	$(100+0.05(100))=\$105$	$100+0.05(100)=\$105$
$t=2$	$100(1+2(0.05))=\$110$	$100(1+0.05)^2=\$110.25$
$t=3$	$100(1+3(0.05))=\$115$	$100(1+0.05)^3=\$115.76$
$t=4$	$100(1+4(0.05))=\$120$	$100(1+0.05)^4=\$121.55$
$t=5$	$100(1+5(0.05))=\$125$	$100(1+0.05)^5=\$127.63$
$t=100$	$100(1+100(0.05))=\$600$	$100(1+0.05)^{100}=\$13{,}150.13$

Compound Interest (once per year): $\boxed{A=P(1+r)^t}$

Ex 2: How much must you invest at age 40 so that you will have a million dollars by the time you retire at 70? Assume an interest rate of 7% compounded annually.

$r=0.07$, $P=?$, $t=30$, $A=\$1{,}000{,}000$

$1000000=P(1+0.07)^{30}$

$\frac{1000000}{1.07^{30}}=P$

$\boxed{P=\$131{,}367.12}$ one lump sum deposit

In general, the formula for compound interest is $\boxed{A = P\left(1+\frac{r}{n}\right)^{nt}}$.

A = balance after t years

P = principal

r = annual interest rate

t = number of years

n = number of times it is compounded per year

$A = P\left(1+\frac{r}{n}\right)^{nt}$

Ex 3: Show the difference between compounding one time per year and twelve times per year when investing $1000 at 5% interest for 10 years.

$P = 1000,\ r = 0.05,\ t = 10$

① $n = 1$

$A = 1000\left(1+\frac{0.05}{1}\right)^{1(10)}$

$\simeq \$1,628.89$

② $n = 12$

$A = 1000\left(1+\frac{0.05}{12}\right)^{10(12)}$

$\simeq \$1,647.01$

Ex 4: What if the compounding on example 3 is continuous?

$r = 0.05,\ P = 1000,\ t = 10$

$A = 1000\left(e^{0.05(10)}\right)$

$\simeq \$1,648.72$

Formula for continuous compounding:

$\boxed{A = Pe^{rt}}$

Exponential Growth and Decay

$\boxed{A(t) = A_0 e^{kt}}$ is the general formula for the exponential growth or decay of a substance.

$k > 0$ growth ; $k < 0$ decay

A_0 read "A naught" (value of A when $t=0$)

Ex 5: The Half-life of radium (^{226}R) is 1620 years. What percent of the radium will still be present after 150 years?

step ① compute k.

$$\frac{1}{2}\cancel{A_0} = \cancel{A_0} e^{1620k}$$

$$\frac{1}{2} = e^{1620k}$$

$$\ln\left(\frac{1}{2}\right) = 1620k$$

$$k = \frac{\ln\left(\frac{1}{2}\right)}{1620} \simeq -0.000428$$

step ②

$$A = A_0 e^{-0.000428t}$$

$A = ?$ when $t = 150$ yrs

$$A = A_0 e^{-0.000428(150)}$$

$$\simeq 0.938 A_0$$

$\Rightarrow$ 93.8%

Ex 6: A certain strain of dangerous bacteria is known to grow from 1000 to 5000 in 5 hours. Assume it grows according to the formula above. $A = A_0 e^{kt}$

points (t, A) ① $(0, 1000)$ ② $(5, 5000)$

a) Determine k, the growth constant and find a formula for the growth, $A(t)$.

(expect $k > 0$) $A_0 = 1000 \Rightarrow A = 1000e^{kt}$

use pt ② to find k.

$$5000 = 1000e^{k(5)}$$

$$5 = e^{5k}$$

$$\ln 5 = 5k$$

$$k = \frac{\ln 5}{5} \simeq 0.32189$$

b) When will the number present be 12,000?

$t = ?$ when $A = 12000$ $A = 1000e^{0.32189t}$

$$12000 = 1000e^{0.32189t}$$

$$12 = e^{0.32189t}$$

$$\ln 12 = 0.32189t$$

$$t = \frac{\ln 12}{0.32189} \simeq \boxed{7.72 \text{ hours}}$$

Ex 7: A car that is priced at $25,000 new, is worth $15,000 after two years.

a) Find the linear model of depreciation. $V = mt + \boldsymbol{b}$

b) Find the exponential model of depreciation. $V = ae^{kt}$

c) Sketch a graph of the two models.

d) Determine the value of the car at the end of five years for each of the models.

points (t, V)

① $(0, 25000)$

② $(2, 15000)$

(a) $V = mt + 25000$

$m = \dfrac{25000 - 15000}{0 - 2} = -5000$

$\boxed{V = -5000t + 25000}$

(b) $V = ae^{kt}$

$a = 25000$

$(a = V_0)$

$V = 25000e^{kt}$

$15000 = 25000e^{k(2)}$

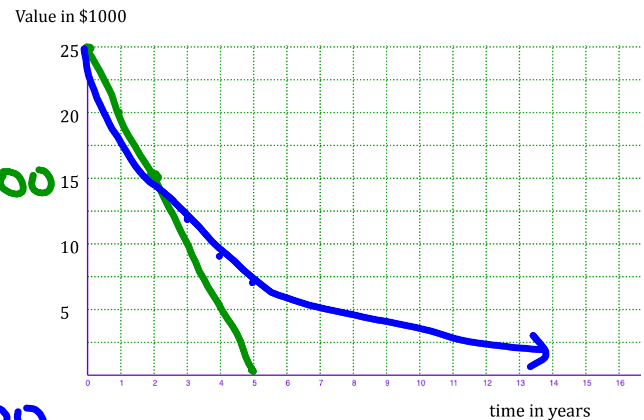
$0.6 = e^{2k}$

$\ln 0.6 = 2k$

$k = \frac{1}{2}\ln 0.6$

$k \simeq -0.2554$

$\boxed{V = 25000e^{-0.2554t}}$



line: $V = -5000t + 25000$

exp. curve: $V = 25000e^{-0.2554t}$

(d) $V = -5000(5) + 25000$

line: $V = \$0$

exp: $V = 25000e^{-0.2554(5)}$

$V \simeq \$6{,}971.37$