8.3 The inverse of a square matrix

You will:

- Verify that matrices are inverses of each other.
- Determine the inverse of a 2x2 matrix if it exists.
- Use Gauss-Jordan elimination to determine the inverse of a 3x3 matrix.
- Use inverse matrices to solve systems of linear equations.

Let A be an *n x n* matrix and I_n be the *n x n* identity matrix. If there exists a matrix A^{-1} such that

$$A\,A^{-1} = I_n = A^{-1}A$$

then A^{-1} is called the inverse of A. The symbol A^{-1} is read *A inverse.*

Example 1:

Which two are inverses?

$$A = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

Process for finding A^{-1} :

Augment A with I
Perform row operations until the left side of the augmented matrix looks like I.
The right side is A^{-1}

Example 2:

a) Find the inverse if it exists:

$$\begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$$

b) Find the inverse if it exists:

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 4 & 0 \\ 2 & 5 & 5 \end{bmatrix}$$

Using Matrix Algebra to solve systems of linear equations.

Example 3:

a) Find A^{-1}:

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix}$$

b) Use the inverse above to solve this system of equations:

If A X = B, then $X = A^{-1}B$

$$\begin{aligned} x + 2y + 2z &= 0 \\ 3x + 7y + 9z &= 1 \\ -x - 4y - 7z &= 2 \end{aligned}$$