Composition of functions

Inverse functions

Today's objectives

- Define composition of functions
- Give examples of composing functions algebraically and by graphing
- Define inverse function
- Practice finding inverse function algebraically and by graphing

Beads and necklaces

- Few years ago I took up beading for fun. I would buy a bag of varied beads and found that I can make 14 necklaces from it.

$f(b)$: # of necklaces I make from b bags of beads

$$f(b) = 14b$$

- As my beading skills got better, I found that people liked my designs and are willing to pay for my necklaces. I started selling them at a local farmers' market for $9.50.

$g(n)$ = earnings from n necklaces

$$g(n) = 9.5n$$

- I would like to know how much money I will make based on the number of bags of beads I buy.

$$\begin{tikzcd}
b \arrow[rr, mapsto, "g \circ f"] \arrow[dr, "f"'] & & \text{earnings } g(f(b)) \\
& n = f(b) \arrow[ur, "g(n)"'] &
\end{tikzcd}$$

Definition

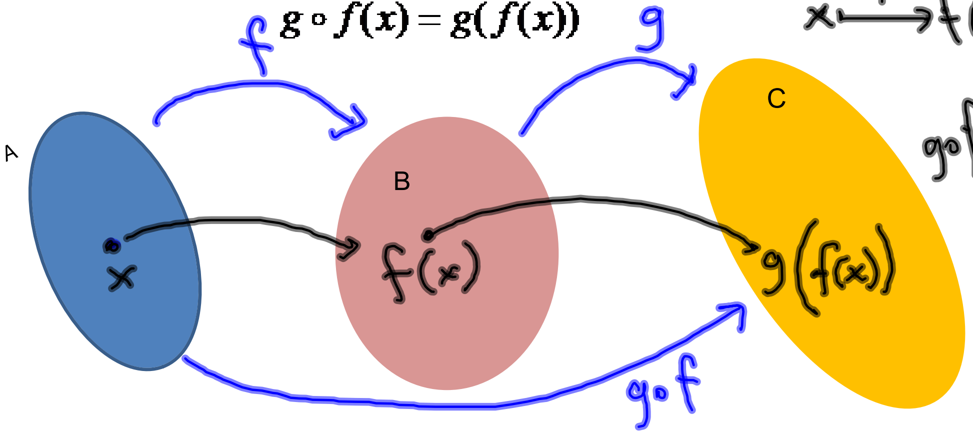
- Let *f: A* $\longrightarrow$ *B, g: B* $\longrightarrow$ *C* be two functions.

Composition of *f* and *g* is a function, denoted by $g \circ f$, defined by:

$$g \circ f(x) = g(f(x))$$

$$x \xrightarrow{f} f(x) \xrightarrow{g} g(f(x))$$

$$g \circ f : A \longrightarrow C$$



Find $g_{o}f$ if

$f(x) = 7x - 2$
$g(x) = x^2 - 2x$

$g \circ f(x) = g(f(x)) = g(7x-2) =$
$= (7x-2)^2 - 2(7x-2) =$
$= 49x^2 - 28x + 4 - 14x + 4 =$
$= 49x^2 - 42x + 8$

$f(x) = x^2 - 2x$
$g(x) = 7x - 2$

$g \circ f(x) = g(f(x)) = g(x^2 - 2x) =$
$= 7(x^2 - 2x) - 2 =$
$= 7x^2 - 14x - 2$

$g \circ f \neq f \circ g$

The following functions can be written as $g \circ f$. What are f and g?

$$F(x) = \sqrt{x^2 - 2x + 1}$$

$g(x) = \sqrt{x}$

$f(x) = x^2 - 2x + 1$

$g \circ f(x) = g(f(x)) = g(x^2 - 2x + 1) = \sqrt{x^2 - 2x + 1} = F(x)$

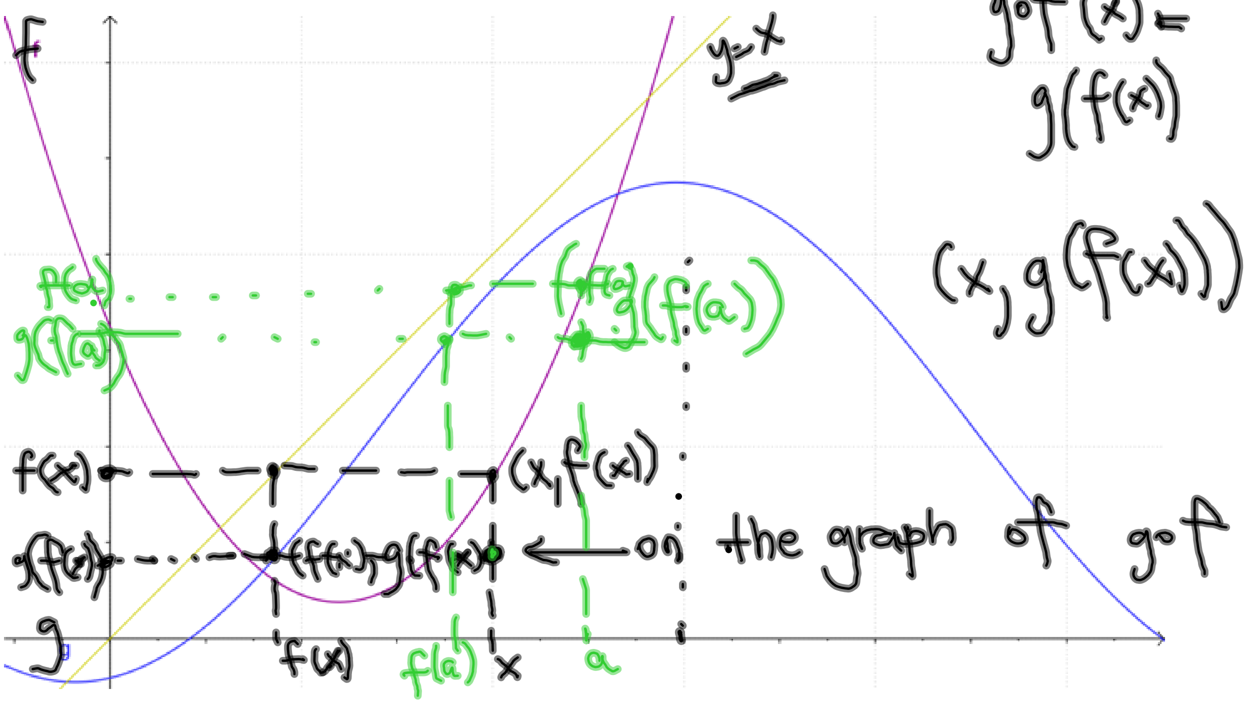
$$F(x) = \frac{x+2}{x+7} = \frac{x+2}{(x+2)+5}$$

$g(x) = \frac{x}{x+5}$

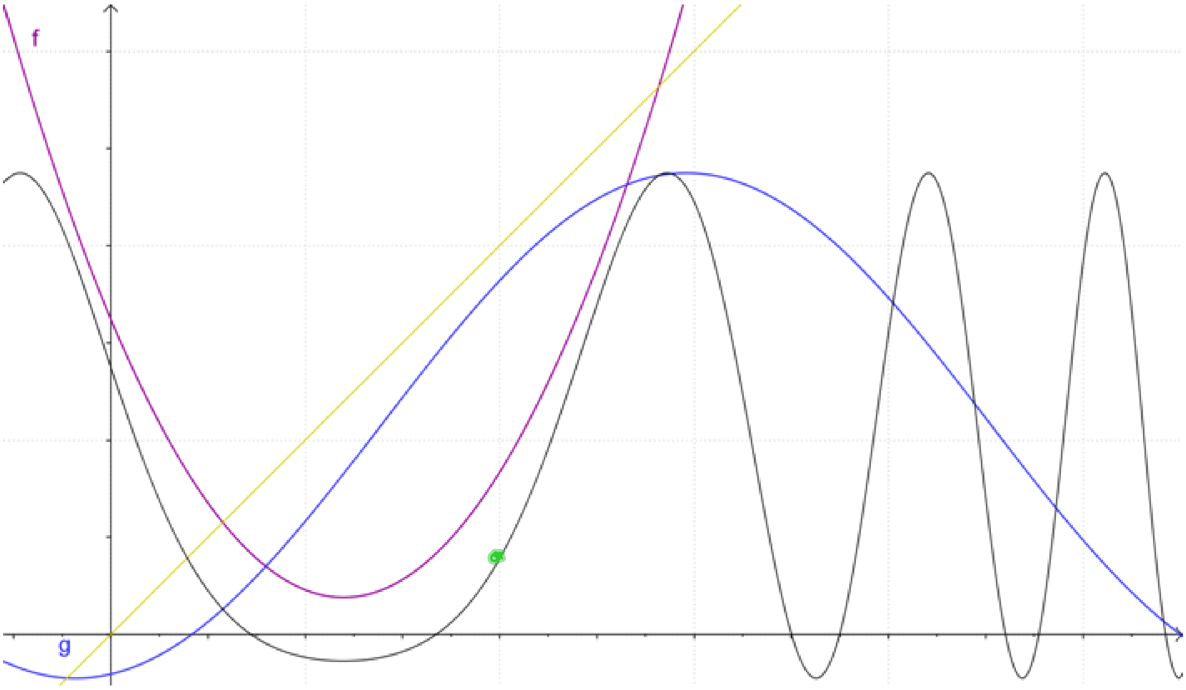
$f(x) = x+2$

$g \circ f(x) = g(f(x)) = g(x+2) = \frac{x+2}{x+2+5} = \frac{x+2}{x+7} = F(x)$

Graphing composition of functions



If we did a whole bunch of points



Remember my beading problem?

- As my beading skills got better, I found that people liked my designs and are willing to pay for my necklaces. I started selling them at a local farmers' market for $9.50.

$$g(n) = 9.5n$$

- I would like to know how many necklaces I need to make in order to earn $779.

$$779 = 9.5n \quad / \div 9.5$$

$$\frac{779}{9.5} = n$$

$$82 = n$$

What if I wanted represent n in terms of my earnings

$$g(n) = 9.5n \quad / \div 9.5$$

$$\frac{g(n)}{9.5} = n$$

$$g(n) = m$$

$$\frac{m}{9.5} = n$$

$h(m)$ = # necklaces needed to make \$ m

$$h(m) = \frac{m}{9.5}$$

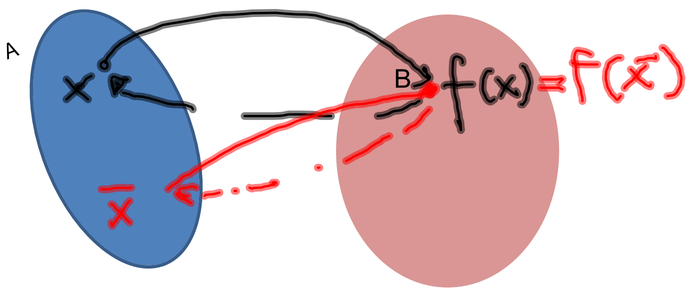
$$\begin{tikzcd} n \arrow[r, "g"] & m \arrow[l, bend left, "h"] \end{tikzcd}$$

$$h(g(m)) = m$$

$$g(h(m)) = m$$

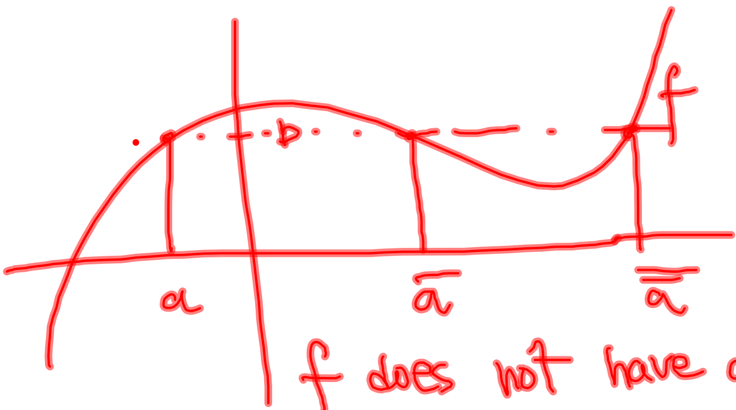
Interesting question

- If I have a function *f* can I find function *g* so that $g \circ f(x) = x$?



x	$f(x)$	
a	b	
$\bar{a}$	b	×

x	$g(x)$	
b	a	
b	$\bar{a}$	×



f does not have an inverse

f does not pass horizontal line test

Inverse function

- If a function *f: A* $\rightarrow$ *B* has the property that each element of *B* is the image of exactly one element of *A* (we say *f* is *injective*), then *f* has an *inverse function,* f^{-1}

$$f \circ f^{-1}(x) = x$$
$$f^{-1} \circ f(x) = x$$

- *Horizontal line test:* Function *f* has an inverse if each horizontal line intersects the graph of *f* in **exactly** one point.

Finding the inverse function

x	$f(x) = y$
1	4
2	5
3	6

x	$f^{-1}(x)$
4	1
5	2
6	3

$y = f(x)$ $\rightarrow$ solve for x so that you have x in terms of y

Finding expression for inverse

$f(x) = 2x+1$

$y = 2x+1 \quad / -1$

$y-1 = 2x \quad / \div 2$

$\frac{y-1}{2} = x$

$f^{-1}(x) = \frac{x-1}{2}$

$g(x) = \frac{x-3}{2x+1}$

$\frac{x-3}{2x+1} = y \quad / (2x+1)$

$x-3 = y(2x+1)$

$x-3 = 2xy+y$

$x-2xy = y+3$

$x(1-2y) = y+3 \quad / \div (1-2y)$

$x = \frac{y+3}{1-2y}$

$g^{-1}(x) = \frac{x+3}{1-2x}$

$h(x) = 2x^2+1$

$h(x) = 2x^2+1$

$y = 2x^2+1 \quad / -1$

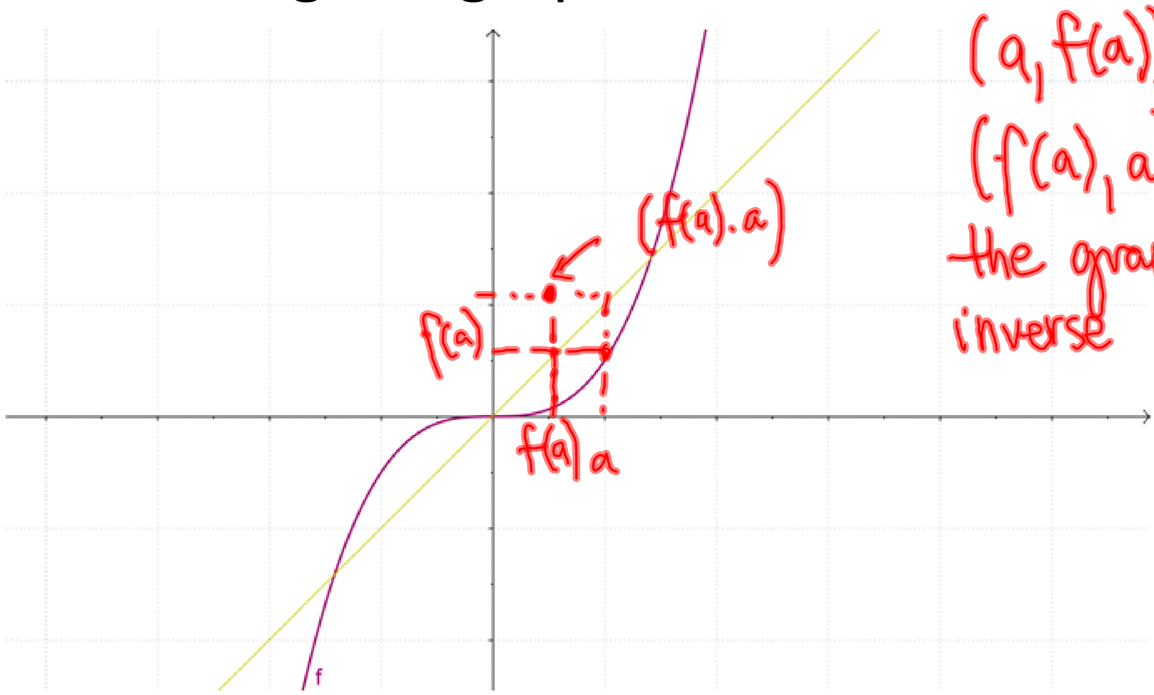
$y-1 = 2x^2 \quad / \div 2$

$\frac{y-1}{2} = x^2 \quad / \sqrt{\ }$

$\pm\sqrt{\frac{y-1}{2}} = x$

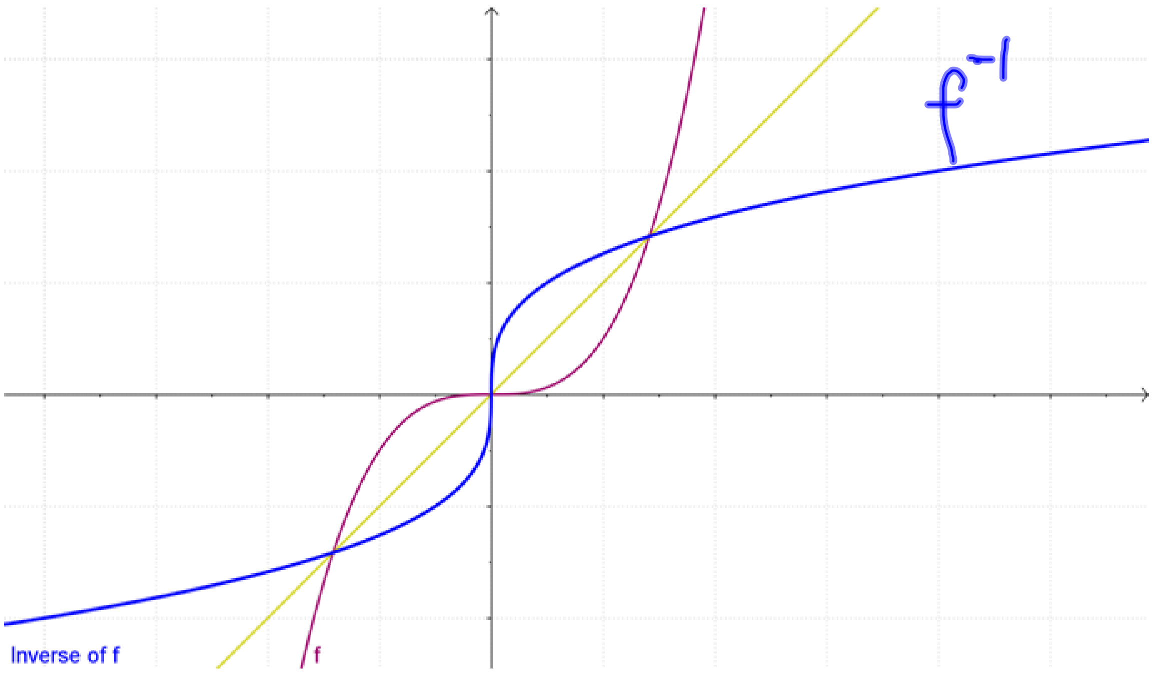
h does not have an inverse

Finding the graph of inverse function



(a, f(a))

(f(a), a) on the graph of inverse



f^{-1}
Inverse of f
f