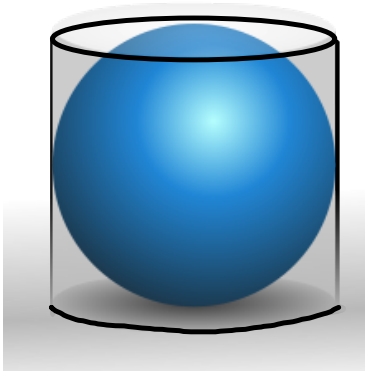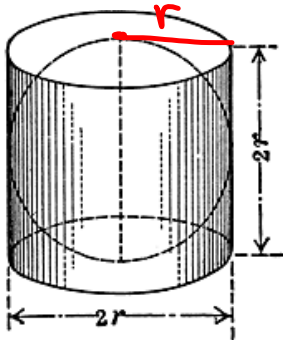Extra Geometry Challenge

Archimedes has a special sphere. He wants to build a cylinder to put it in so that it just fits. The sphere is to be painted with a very expensive paint. The cylinder will be painted with an even more expensive paint, so he wants his purchase to be precise. If the sphere uses exactly four quarts of paint, how much paint will he have to purchase to paint the cylinder (including the top and bottom)?

Extra Geometry Challenge Solution

Sphere uses 4 quarts of paint. How much is needed for the cylinder?

$r = $ radius of sphere (& cylinder)

$$S_{cyl} = 2\pi r^2 + 2\pi r h$$
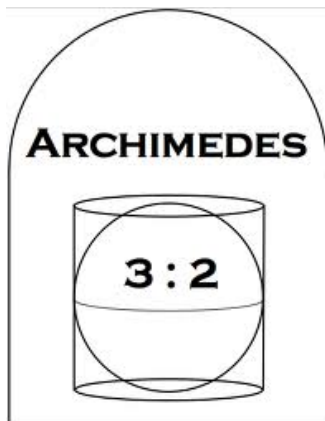$$= 2\pi r^2 + 2\pi r (2r)$$
$$= 2\pi r^2 + 4\pi r^2 = 6\pi r^2$$

$$S_{sphere} = 4\pi r^2$$

$$\Rightarrow \frac{S_{cyl}}{S_{sphere}} = \frac{6\pi r^2}{4\pi r^2} = \frac{6}{4} = \frac{3}{2}$$

$$\Rightarrow S_{cyl} = \frac{3}{2} S_{sphere}$$

$\Rightarrow$ cylinder needs $\frac{3}{2}(4 \text{ qts}) = 6$ qts of paint

Final question: How do the volumes of these two compare? Why do you think Archimedes wanted this on his tombstone?

ARCHIMEDES

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$$V_{sphere} = \frac{4}{3}\pi r^3$$

$$V_{cyl} = \pi r^2 h = \pi r^2 (2r)$$
$$= 2\pi r^3$$

$$\frac{V_{cyl}}{V_{sphere}} = \frac{2\pi r^3}{\frac{4}{3}\pi r^3} = \frac{2}{4/3} = 2 \cdot \frac{3}{4} = \frac{3}{2}$$