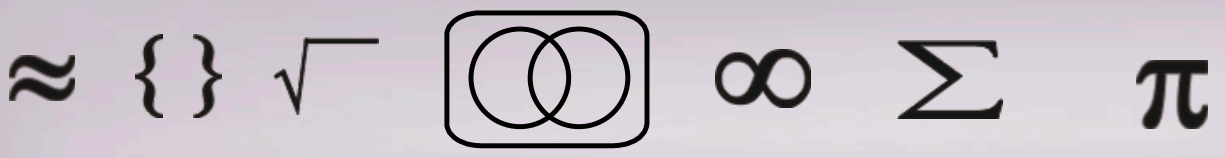

APR, APY

Math 1030 # 8b

compound interest

The Power of Compounding

effective yield

Continuous Interest, APY

principal

balance

Continuous Compounding means they are compounding an infinite number of times per year. Here is the formula.

$A = P \cdot e^{(APR \cdot Y)}$ often seen as $A = Pe^{rt}$

EX 1: Find the balance after 1, 5, and 20 years if you invest $500 in an account with 2.7% APR compounded continuously.

$P = \$500$, $APR = 0.027$

$Y = 1$: $A = 500e^{0.027(1)} \approx \$513.68$

$Y = 5$: $A = 500e^{0.027(5)} \approx \$572.27$

$Y = 20$: $A = 500e^{0.027(20)} \approx \$858.00$

Some questions:

- How does the APR affect the balance?
  higher APR ⇒ higher balance
- How does the amount of time affect the balance?
  the more time, the better; w/ MUCH greater gains at the end
- How does the number of compounding times per year affect it?
  more frequent compounding increases acct. balance, but the difference above compounding daily (or even monthly) is not that great

Annual Percentage Yield (APY) is the actual percentage by which the balance increases in one year (the relative increase in 1 year).

- If compounded annually, APY = APR.
- If compounded more than once per year, APY > APR.
- The APY does not depend on the starting principal.
- APY is also called effective yield or the yield.

EX 2: Find the annual percentage yield to the nearest hundredth of a percent if a bank offers an APR of 2.25% compounded quarterly.

Let's say we have P principal

$$A = P\left(1+\frac{APR}{n}\right)^{nY}$$

relative change in one yr $\Rightarrow$ plug in $Y=1$

for $Y=1$:

$$A = P\left(1+\frac{APR}{n}\right)^{n}$$

$$\text{rel. change} = \frac{P\left(1+\frac{APR}{n}\right)^{n}}{P} = \left(1+\frac{APR}{n}\right)^{n}$$

for our example: $APR = 0.0225$, $n=4$

$$\text{rel. change} = \left(1+\frac{0.0225}{4}\right)^{4} \simeq 1.022690$$

$$= 1 + 0.022690$$

$\Rightarrow APY \simeq 0.02269$

$= 2.269\%$

$\Rightarrow APY =$ relative change of acct in 1 yr $- 1$

final answer:

2.27%

$$APY = \left(1+\frac{APR}{n}\right)^{n} - 1$$

EX 3: How much must you deposit today so that you have $180,000 (for a college fund) in 11 years? Assume no additional deposits will be made in an account which pays an APR of 6% and compounds monthly.

$A = 180{,}000$, $APR = 0.06$, $n = 12$, $Y = 11$

$$A = P\left(1 + \frac{APR}{n}\right)^{nY}$$

$$180000 = P\left(1 + \frac{0.06}{12}\right)^{(12(11))}$$

$$180{,}000 \simeq P(1.9316)$$

$$P \simeq \$93{,}186.36$$