Linear Function

Math 1030 #14b

Linear Equation

Linear Modeling

Rate of Change

The Equation of a Line

Price-Demand Function

You begin the year with 25 songs on your playlist and add 6 songs each month. A way to express how many songs you will have in any future month is

number of songs = 25 songs + (6 songs/month)(# of months)

rate of change

If we let N = *the number of songs* and t = *time in months*,

$N = 25 + 6t$

This gives us a <u>general equation for a linear function.</u>

dependent variable = initial value + (rate of change)(independent variable)

or

$y = b + mx$, where m is the slope (rate of change) and b is the initial value.

We generally write it this way: $y = mx + b$ (slope-intercept form)

How to write the equation of a line

Let x = the independent variable

    y = the dependent variable

Step 1: Determine the slope (rate of change) of the line.

$$m = \frac{change\ in\ x}{change\ in\ y} = \frac{y_2 - y_1}{x_2 - x_1}$$

Step 2: To find the initial value, substitute m, x and y into the equation and solve for b.

Step 3: Write the equation, $y = mx + b$, using the values found above.

EX 1: Write an equation for the line which contains these points:

(3,5) and (6,14)

(x_1, y_1) (x_2, y_2)

① $m = \frac{14-5}{6-3} = \frac{9}{3} = 3$

$\left(m = \frac{5-14}{3-6} = \frac{-9}{-3} = 3 \right)$

② find b:

$y = mx + b$

choose pt (3,5) to plug in

$5 = 3(3) + b$

$5 = 9 + b$

$-4 = b$

③ $\boxed{y = 3x - 4}$

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EX 2: You can purchase a motorcycle for $6500 or lease it for a down payment of $200 and $150 payment each month.

t = input (indep. var.), C = output (dep. var.)

a) Find an equation that describes how the cost (C, *measured in dollars*) of the lease depends on the time (t, *measured in months*).

$$C = mt + b \qquad b = \$200$$
$$m = \$150/mo$$

$$\boxed{C = 150t + 200}$$

b) How much have you paid after one year?

$C = ?$ when $t = 12$ mos.

$$C = 150(12) + 200 = \boxed{\$2000}$$

c) How long can you lease it before you've paid more than its purchase price?

purchase price = $\$6500$

$t = ?$ when $C = 6500$

$$C = 150t + 200$$
$$6500 = 150t + 200$$
$$6300 = 150t$$
$$42 = t$$

$\Rightarrow$ after 42 months (or $3\frac{1}{2}$ yrs)

you've paid the entire purchase price.

EX 3: Two astronauts on an outer space mission love dehydrated astronaut ice cream sandwiches. This table shows how many sandwiches are left on any given day of the mission, starting with the beginning of the trip.

t	1	3	6	10
Q	96	88	76	60

t = # of days they've been on the mission

Q = # of sandwiches left

$$Q = mt + b$$

a) Write a linear equation for this data.

note: "(x,y)" = (t, Q)

① find m: $(1, 96)$ $(3, 88)$
(x_1, y_1) (x_2, y_2)

$$m = \frac{88-96}{3-1} = \frac{-8}{2} = -4$$ (−4 means there are 4 fewer sandwiches per day)

② find b: plug in m=−4, t=1, Q=96
$$96 = -4(1) + b \iff b = 100$$

③ eqn of line: $\boxed{Q = -4t + 100}$

b) On what day of the mission will there be only 4 sandwiches left?

$t = ?$ when $Q = 4$

$$4 = -4t + 100$$
$$\frac{-96}{-4} = \frac{-4t}{-4}$$
$$24 = t$$

(assuming they continue eating 4 ice cream sandwiches per day)