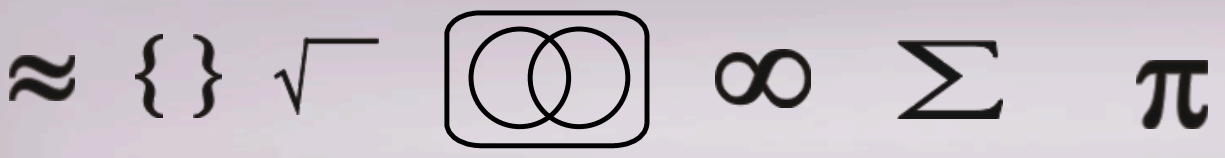

Math 1030 #11a

Doubling

Absolute Growth

Linear Growth vs Exponential Growth

Relative growth

The Concept and Examples

Basic Growth Patterns

Linear growth -- when a quantity grows by the same absolute amount in each unit of time.

Exponential growth -- when a quantity grows by the same relative amount in each unit of time.

EX 1: Identify the growth pattern in each of these as linear, exponential or neither. Then answer the question.

a) Your allowance for each week starts with $1.00 the first day, $1.00 the second day, $2.00 on the third day, then $3.00 on the fourth day, each day's allowance being the sum of the two previous days'. The pattern starts over each week. How much will you have after 2 weeks?

neither linear nor exponential

$2(1+1+2+3+5+8+13) = \$66$

b) The value of your house is decreasing by 7% per year. If it is worth $250,000 today, what will it be worth in five years?

exponential "growth"

$250{,}000(0.93)(0.93)(0.93)(0.93)(0.93)$

$= 250{,}000(0.93)^5 \simeq \$173{,}922.09$

c) The price of a gallon of gas is increased by 3 cents per week. If the price is $3.10/gal today, what will it be in ten weeks?

linear growth

$3.10 + \underbrace{0.03 + 0.03 + 0.03 + \ldots + 0.03}_{10 \text{ times}}$

$= 3.10 + 0.03(10) = \$3.40/\text{gal}$

EX 2: A rare snake gives birth to exactly 3 babies and expires immediately. Those snakes each give birth to 3 more after six months, expiring immediately.

Zootown starts with 15 of these snakes.

a) Fill in this table to answer the questions below.

n	# months	# snakes
0	0	15
1	6	$15(3)=45$
2	12	$45(3)=15(3^2)=135$
3	18	$15(3^2)(3)=15(3^3)=405$
4	24	$15(3^4)=1215$
5	30	$15(3^5)=3645$

n = number of 6-month increments

b) How many snakes are there in 16 years?

16 yrs = 32 6mo increments

$\Rightarrow$ # snakes $= 15(3^{32}) \approx 2.77953 \times 10^{16}$

c) If Zootown has room for only 250 of these snakes, when do they have to start selling them to other zoos?

n = ? when number of snakes is 250

$250 = 15(3^n)$

$\Rightarrow$ between 12 and 18 months

EX 3: You have so many Facebook friends to keep track of, that you decide to pare down your list to 15 friends. You allow yourself to add only 3 friends each six months.

(this is linear growth)

a) Make a table of the total number of contacts you have.

n	# months	# Friends
0	0	15
1	6	$15+3=18$
2	12	$15+3+3=15+2(3)=21$
3	18	$15+2(3)+3=15+3(3)=24$
4	24	$15+4(3)=27$
5	30	$15+5(3)=30$

b) How long will it take to get 250 friends?

$n=?$ when # friends $=250$

$250=15+n(3)$

$-15 \quad -15$

$235=3n$

$n=\frac{235}{3}=78\frac{1}{3}$

$\Rightarrow$ it takes over 78 6-month increments = just over 39 yrs.

c) How many total friends will there be after 16 years?

16 yrs = 32 6-month increments

$\Rightarrow n=32$

# of friends $=15+n(3)$

$=15+32(3)=111$

EX 4: Let's compare the tables in the two preceding examples.

exp growth lin. growth

months	snakes	Facebook contacts
0	15	15
6	45	18
12	135	21
18	405	24
24	1215	27
30	3645	30
36	10,935	33