MATH 1010 ~ Intermediate Algebra

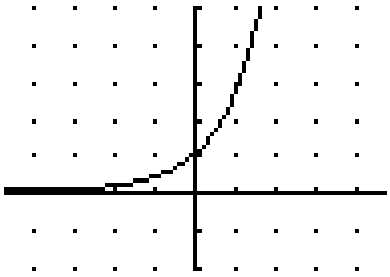
Chapter 9: EXPONENTIAL AND LOGARITHMIC FUNCTIONS

# Section 9.1: Exponential Functions

Objectives:

- Evaluate exponential functions.
- Graph exponential functions.
- Define the natural base *e* and graph natural exponential functions.
- Use the natural base *e* in an application.

$$f(x) = e^x$$



# Exponential Functions: fn w/ variable in the exponent

Defn

$f(x) = a^x \qquad a > 0 \qquad a \neq 1$

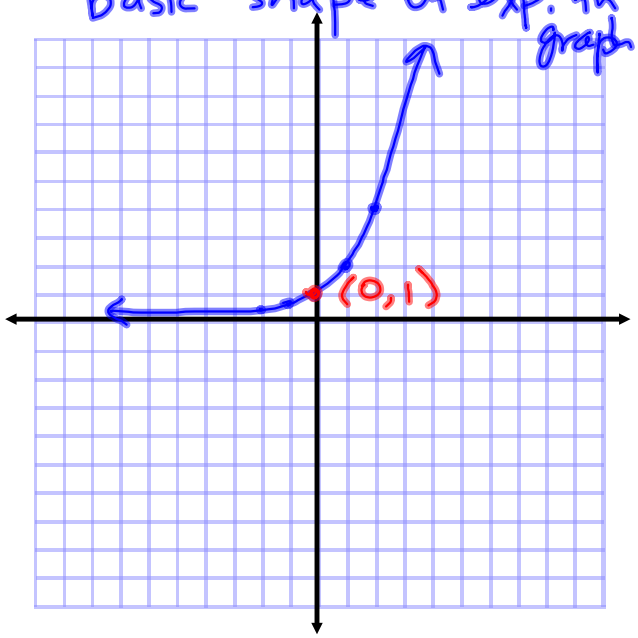
a constant

| $f(x) = a^x$ | $f(x) = x^a$ |
|---|---|
| ex $f(x) = 2^x$ | ex $f(x) = x^2$ |
| exponential | power/polynomial |

$y = 2^x$

| x | y | |
|---|---|---|
| 0 | 1 | $y = 2^0$ |
| 1 | 2 | $y = 2^1$ |
| 2 | 4 | $y = 2^2$ |
| -1 | ½ | $y = 2^{-1}$ |
| -2 | ¼ | $y = 2^{-2}$ |

basic shape of exp. fn graph



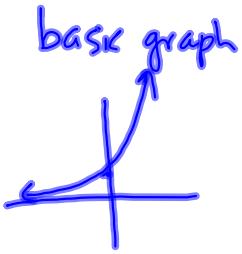

HA (horizontal asymptote)

curve never touches or crosses HA

$y = 0$ HA (for every basic exponential curve)

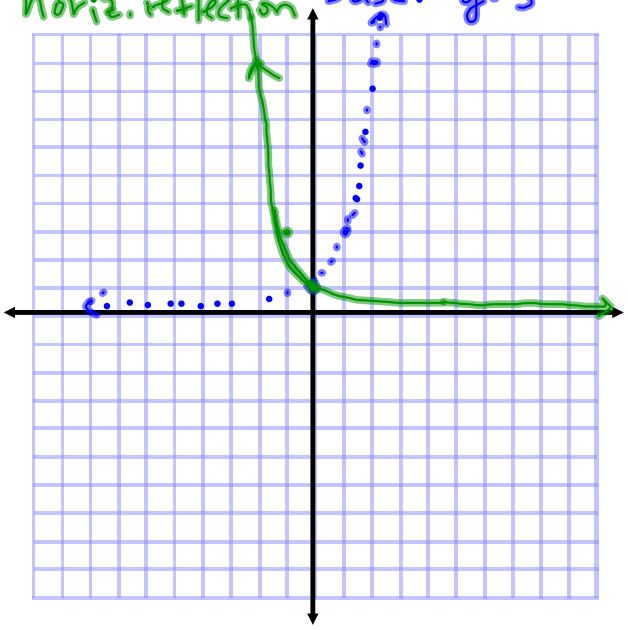
① EXAMPLE

Sketch these using transformations of $y = a^x$




a) $y = 3^{-x}$

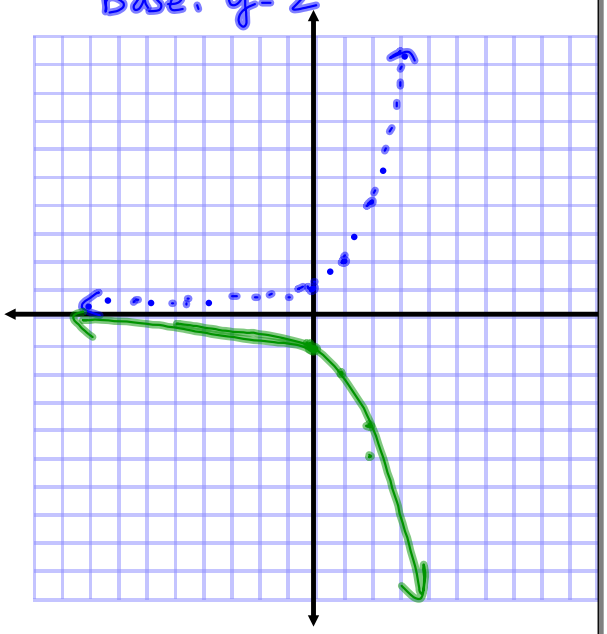
horiz. reflection base: $y = 3^x$



b) $y = -2^x$

reflection: vertical

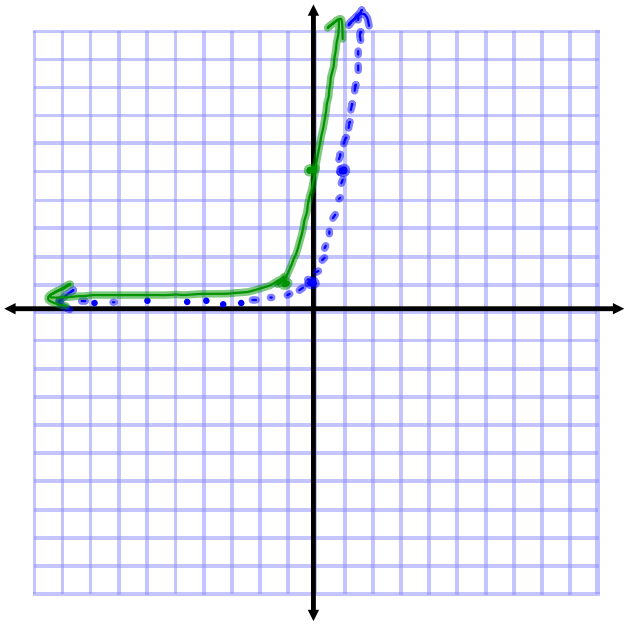
base: $y = 2^x$



c) $y = 5^{x+1}$

base: $y = 5^x$

horizontal shift: left 1

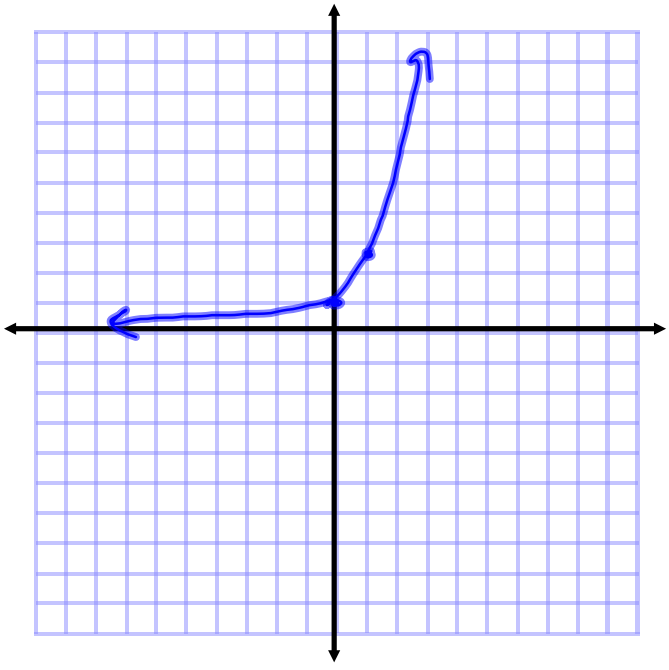


Introducing a new constant.... meet $e$ .

e irrational number (in decimal form, the number never ends and doesn't repeat)

$e \simeq 2.718$

$y = e^x$ natural exponential fn



② EXAMPLE

Simplify these expressions.

a) $\sqrt{4e^{6x}} = \sqrt{4}\sqrt{e^{6x}}$

$= 2(e^{6x})^{1/2}$

$= 2e^{3x}$

b) $\frac{6e^5}{10e^7} = \frac{3e^{5-7}}{5}$

$= \frac{3e^{-2}}{5}$

$= \frac{3}{5e^2}$

c) $(e^3)^2 = e^6$

d) $e^{2x}e^{-3x}$

$= e^{2x-3x}$

$= e^{-x}$ or $\frac{1}{e^x}$

③ EXAMPLE

Evaluate these functions at the given value.

$a)\quad g(x) = 10{,}000(1.03)^{4x}$

$g(1) = 10{,}000(1.03)^{4(1)} = 10000\,(1.03^4)$

$\cong 10000(1.1255)$

$= 11255.0881$

WARNING: $\neq (10000(1.03))^4$

$g(3) = 10000(1.03^{12}) \simeq 10000(1.425760887)$

$= 14257.60887$

$b)\quad P(t) = \dfrac{6000}{2 + e^{0.05t}}$

$P(2) = \dfrac{6000}{2+e^{0.05(2)}} = \dfrac{6000}{2+e^{0.1}} \simeq \dfrac{6000}{2+1.105}$

$\simeq 1932.26$

$P(0) = \dfrac{6000}{2+e^{0.05(0)}} = \dfrac{6000}{2+e^{0}} = \dfrac{6000}{2+1}$

$= \dfrac{6000}{3} = 2000$

# APPLICATION

$y$ = amt/value after $t$ yrs

$$y = Pe^{rt}$$

$t$ = time (yrs)

$r$ = interest rate (annual)

$P$ = principal

When your child is born you deposit $5000 in an account that pays 3% continuously compounded interest. How much will be there when the child turns 18?

$P = \$5000 \quad r = 0.03 \quad t = 18$ yrs.

$$y = 5000(e^{0.03(18)})$$

$$\approx \$8580.03$$