MATH 1010 ~ Intermediate Algebra

Chapter 6: RATIONAL EXPRESSIONS, EQUATIONS AND FUNCTIONS

Section 6.4: Complex Fractions

Objectives:

- ✯ Simplify complex fractions using rules for dividing rational expressions.
- ✯ Simplify complex fractions having a sum or difference in the numerator and/or denominator.

$$\frac{3/2}{4x+1}$$

$$\frac{\frac{x-2}{x+4}}{\frac{x+3}{x-2}}$$

$$\frac{4+\frac{16}{x-4}}{5+\frac{20}{x-4}}$$

Simplify these:

a) $\dfrac{\left(\dfrac{3u^2}{6v^3}\right)}{\left(\dfrac{u}{3v}\right)}$ $= \dfrac{3u^2}{6v^3} \div \dfrac{u}{3v}$ $v \neq 0, u \neq 0$

$= \dfrac{\cancel{3}u^{\cancel{2}}}{\cancel{6}_2 v^{\cancel{3}\,2}} \cdot \dfrac{3\cancel{v}}{\cancel{u}}$

$= \boxed{\dfrac{3u}{2v^2}, \; u \neq 0}$

b) $\dfrac{\left(\dfrac{x}{x-4}\right)}{\left(\dfrac{x}{4-x}\right)}$ $= \dfrac{x}{x-4} \div \dfrac{x}{4-x}$ $x \neq 0, 4$

$= \dfrac{x}{x-4} \cdot \dfrac{4-x}{x}$ $4 - x = -x + 4$
$= -(x-4)$

$= \dfrac{\cancel{x}(-1)\cancel{(x-4)}}{\cancel{(x-4)}\cancel{x}} = \boxed{-1, x \neq 0, 4}$

$\left(\dfrac{\dfrac{x}{x-4}}{\dfrac{x}{4-x}}\right) = \left(\dfrac{\dfrac{x}{(x-4)}}{\dfrac{-x}{(x-4)}}\right)\left(\dfrac{\dfrac{\cancel{(x-4)}}{1}}{\dfrac{\cancel{(x-4)}}{1}}\right)$ $LCD = x - 4$

$= \dfrac{\dfrac{x\cancel{(x-4)}}{\cancel{(x-4)}}}{\dfrac{-x\cancel{(x-4)}}{\cancel{(x-4)}}} = \dfrac{x}{-x} = \boxed{-1, \; x \neq 0, 4}$

c) $\dfrac{\left(\dfrac{x^2-2x-8}{x-1}\right)}{5x-20} = \dfrac{\dfrac{(x-4)(x+2)}{x-1}}{5(x-4)}$

$= \dfrac{(x-4)(x+2)}{(x-1)} \div (5(x-4))$

$= \dfrac{\cancel{(x-4)}(x+2)}{(x-1)} \cdot \dfrac{1}{5\cancel{(x-4)}}$

$= \boxed{\dfrac{x+2}{5(x-1)},\ x \neq 4}$

$x \neq 0, -1, 5/2, -1/5$

$\neq 0$ d) $\dfrac{\left(\dfrac{6x^2-13x-5}{5x^2+5x}\right)}{\left(\dfrac{2x-5}{5x+1}\right)} = \dfrac{\dfrac{(3x+1)(2x-5)}{5x(x+1)}}{\dfrac{(2x-5)}{(5x+1)}}$

$6 \cdot -5 = -30$

$-15 \cdot 2$

	$2x$	-5
$3x$	$6x^2$	$-15x$
1	$2x$	-5

$LCD = 5x(x+1)(5x+1)$

$= \dfrac{\dfrac{(3x+1)(2x-5)}{\cancel{5x(x+1)}}}{\dfrac{(2x-5)}{\cancel{(5x+1)}}} \left(\dfrac{\dfrac{\cancel{5x(x+1)}(5x+1)}{1}}{\dfrac{5x(x+1)\cancel{(5x+1)}}{1}}\right)$

$= \dfrac{(3x+1)\cancel{(2x-5)}(5x+1)}{\cancel{(2x-5)}(5x)(x+1)} = \boxed{\dfrac{(3x+1)(5x+1)}{5x(x+1)},\ x \neq \frac{5}{2}, \frac{-1}{5}}$

e) $$\frac{\left(16-\frac{1}{x^2}\right)}{\left(\frac{1}{4x^2}-4\right)}\left(\frac{4x^2}{4x^2}\right) = \frac{16(4x^2)-\frac{1}{\cancel{x^2}}(4\cancel{x^2})}{\frac{1(\cancel{4x^2})}{\cancel{4x^2}}-4(4x^2)}$$

LCD $= 4x^2$

$x \neq 0$

$\frac{1}{4x^2}-4 \neq 0$

$1-16x^2 \neq 0$

still a denominator

$\Rightarrow 1 \neq 16x^2$

$\frac{1}{16} \neq x^2$

$x \neq \pm\frac{1}{4}$

$$= \frac{64x^2-4}{1-16x^2} = \frac{4(16x^2-1)}{1-16x^2}$$

$16x^2-1$

$= -(1-16x^2)$

$$= \frac{-4\cancel{(1-16x^2)}}{\cancel{(1-16x^2)}}$$

$$= \boxed{-4}\ \boxed{,\ x \neq 0, \pm\frac{1}{4}}$$

$x \neq 0, -2$

f) $$\frac{\left(\frac{x+1}{x+2}-\frac{1}{x}\right)}{\left(\frac{2}{\cancel{x}+2}\right)}\left(\frac{\frac{x(x+2)}{1}}{\frac{x\cancel{(x+2)}}{1}}\right) = \frac{\frac{x(x+1)\cancel{(x+2)}}{\cancel{(x+2)}}-\frac{\cancel{x}(x+2)}{\cancel{x}}}{2x}$$

LCD $= x(x+2)$

$$= \frac{x(x+1)-(x+2)}{2x}$$

$$= \frac{x^2+\cancel{x}-\cancel{x}-2}{2x}$$

$$= \boxed{\frac{x^2-2}{2x},\ x \neq -2}$$

g) $\dfrac{3x^{-2}-x}{4x^{-1}+6x} = \left(\dfrac{\frac{3}{x^2}-x}{\frac{4}{x}+6x}\right)\left(\dfrac{\frac{x^2}{1}}{\frac{x^2}{1}}\right)$

LCD $= x^2$

$x \neq 0$

$\frac{4}{x}+6x \neq 0$

$4+6x^2 \neq 0$

$$= \dfrac{\frac{3\cancel{x^2}}{\cancel{x^2}} - x(x^2)}{\frac{4\cancel{x^2}}{\cancel{x}} + 6x(x^2)} = \dfrac{3-x^3}{4x+6x^3} = \boxed{\dfrac{3-x^3}{2x(2+3x^2)}}$$

h) $\dfrac{x-y}{x^{-2}-y^{-2}} = \dfrac{x-y}{\frac{1}{x^2}-\frac{1}{y^2}}$

LCD $= x^2y^2$

$x \neq 0,\ y \neq 0$

$\frac{1}{x^2}-\frac{1}{y^2} \neq 0$

$\Longrightarrow x \neq \pm y$

$$= \dfrac{x-y}{\frac{1}{x^2}\left(\frac{y^2}{y^2}\right) - \frac{1}{y^2}\left(\frac{x^2}{x^2}\right)}$$

$$= \dfrac{(x-y)}{\frac{y^2-x^2}{x^2y^2}}$$

$$= (x-y) \div \left(\dfrac{y^2-x^2}{x^2y^2}\right) = (x-y)\cdot\left(\dfrac{x^2y^2}{y^2-x^2}\right)$$

$$= \dfrac{(x-y)x^2y^2}{(y-x)(y+x)}$$

Note:
$y-x = -(x-y)$

$$= \dfrac{\cancel{(x-y)}x^2y^2}{-\cancel{(x-y)}(y+x)}$$

still have
$x \neq -y$

$$= \boxed{\dfrac{-x^2y^2}{y+x},\ x\neq 0,\ y\neq 0,\ x\neq -y}$$