MATH 1010 ~ Intermediate Algebra

Chapter 5: POLYNOMIALS AND FACTORING

Section 5.4: Factoring by Grouping and Special Forms

Objectives:

- ◆ Factor the greatest common monomial factors from polynomials.
- ◆ Factor polynomials by grouping.
- ◆ Factor the difference of two squares.
- ◆ Factor the sum and difference of two cubes.
- ◆ Factor polynomials completely.

$$a^3 - b^3 = ?$$

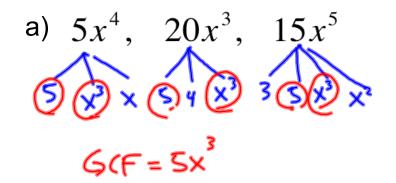
 $x^2 + 2xy + y^2 = ?$

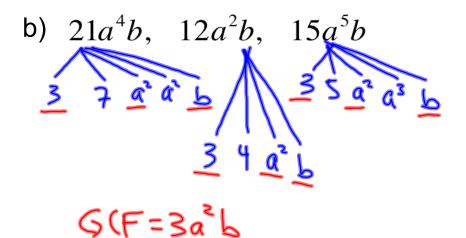
$$a^2 + b^2 = ?$$

GCF: Greatest Common Factor

the largest number (or expression) that goes into all the given terms

Find the GCF.





① EXAMPLE:

Factor out the greatest common factor.

a)
$$24x^3 - 32x^2 = 8x^2(3x - 4)$$

(distributive property "backwards")

b)
$$4x^{2}(3x-1) - 6(3x-1)$$

= $(3x-1)(4x^{2}-6)$
= $(3x-1)(2)(2x^{2}-3)$ or $2(3x-1)(2x^{2}-3)$

grouping c)
$$x^3 - 5x^2 + x - 5$$

= $x^2(x-5) + 1(x-5)$
= $(x-5)(x^2+1)$

d)
$$(3x+7)(2x-1)+(x-6)(2x-1)$$

= $(2x-1)((3x+7)+(x-6))$
= $(2x-1)(3x+7+x-6)$
= $(2x-1)(4x+1)$

<u>Difference of squares</u>

 $a^2 - b^2 = (a - b)(a + b)$

② EXAMPLE Factor these

$$(a-b)(a+b)=a^2+ab-ab-b^2$$

= a^2-b^2

a)
$$9x^2 - 25$$

= $(3x)^2 - 5^2 = (3x - 5)(3x + 5)$
 $4 = 3x, 5 = 6$

b)
$$a^2 - \frac{1}{16} = a^2 - \left(\frac{1}{4}\right)^2 = \left(a - \frac{1}{4}\right)\left(a + \frac{1}{4}\right)$$

$$= \left(a + \frac{1}{4}\right)\left(a - \frac{1}{4}\right)$$

c)
$$(x+3)^2 - 49$$

 $4 = (x+3)^2 - 7^2$
 $4 = (x+3+7)(x+3-7)$
 $4 = (x+10)(x-4)$

Sum and Difference of Cubes

③ Example Factor these.

a)
$$x^{3} - 64$$

= $x^{3} - 4^{3}$ ($x^{2} + 4x + 4^{2}$)
= $(x - 4)(x^{2} + 4x + 16)$
c) $3x^{4} + 81x$
= $3x(x^{3} + 27)$
= $3x(x^{3} + 3^{3})$
= $3x(x^{3} + 3^{3})$

b)
$$8w^3 + 27$$

$$= (2w)^3 + 3^3$$

$$= (2w+3)(2w)^2 - 3(2w) + 3^2$$

$$= (2w+3)(4w^2 - 6w + 9)$$
d) $2a^3 - 32a$

$$= 2a (a^2 - 16)$$

$$= 2a (a^2 - 4^2)$$

$$= 2a (a-4)(a+4)$$

What about the sum of two squares?

$$x^{2} + y^{2}$$
 factors

idea 1: $(x+y)^{2} = x^{2} + y^{2}$?

check: $(x+y)^{2} = (x+y)(x+y)$
 $= x^{2} + xy + xy + y^{2}$