MATH 1010 ~ Intermediate Algebra

Chapter 5: POLYNOMIALS AND FACTORING

Section 5.1: Integer Exponents and Scientific Notation

Objectives:

- Use the rules of exponents to simplify expressions.
- Rewrite exponential expressions involving negative exponents.
- Write very large and very small numbers in scientific notation.

? $4^{-2}$ $\left(\frac{3}{4}\right)^{-1}$ ? ??

? $-3^{-4}$ ?

Review of the rules of exponents

1) $a^m a^n = a^{m+n}$

$2^3 2^4 = 2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2 = 2^7$

2) $a^m \div a^n = a^{m-n}$

3) $(a^m)^n = a^{mn}$

$(2^2)^3 = 2^2\cdot2^2\cdot2^2 = 2^6$

4) $(ab)^m = a^m b^m$

5) $(a/b)^m = \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

6) $a^0 = 1,\ a \neq 0$

7) $a^{-m} = \frac{1}{a^m}$

8) $(a/b)^{-m} = (b/a)^m$

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$a^0 = ?$

$10^3 = 1000$

$10^2 = 100$ $\div 10$

$10^1 = 10$ $\div 10$

$10^0 = 1$ $\div 10$

$10^{-1} = \frac{1}{10}$ $\div 10$

$10^{-2} = \frac{1}{10^2}$ $\div 10$

$10^{-3} = \frac{1}{10^3}$

$3^3 = 27$

$3^2 = 9$ $\div 3$

$3^1 = 3$ $\div 3$

$3^0 = 1$ $\div 3$

$3^{-1} = \frac{1}{3}$ $\div 3$

$3^{-2} = \frac{1}{3^2}$ $\div 3$

what about $0^0$?

$0^3 = 0$ $3^0 = 1$

$0^2 = 0$ $2^0 = 1$

$0^1 = 0$ $\dagger^0 = 1$

$\underbrace{0^0 = 0? \quad 0^0 = 1?}$

$0^0$ undefined

Use the rules to simplify these:

a) $3^2\, x^2 \cdot x^3 =$ $9x^5$

b) $(3x)^2 \cdot x^5 = 3^2 x^2 x^5 = 9x^7$

c) $-(a^3\, b^2\,)^2\,(-ab^3) = -(a^6\; b^4)(-ab^3)$

$= a^6 b^4 a^1 b^3$

$= a^6 a^1 b^4 b^3$

$= a^7 b^7$ or $(ab)^7$

d) $\frac{3x^2 (2x)^2}{(-2x)(6x)} = \frac{\cancel{3}x^2 \cancel{4}\cancel{x^2}}{-\cancel{12}\cancel{x^2}} = \frac{x^2}{-1} = -x^2$

e) $\frac{-1}{6^{-2}} = \frac{-1}{\frac{1}{6^2}} = -1 \div \frac{1}{6^2} = -1 \cdot \frac{6^2}{1} = -6^2 = -36$

f) $(-4^{-1})^{-2} = \left(\frac{-1}{4}\right)^{-2} = \left(\frac{-4}{1}\right)^2$ or $\left(\frac{4}{-1}\right)^2$

$= 16$

g) $(4^0 - 3^{-2})^{-1} = \left(1 - \frac{1}{3^2}\right)^{-1}$

subtraction

Qn: $= \left(1^{-1} - \left(\frac{1}{3^2}\right)^{-1}\right)$

NO

$= \left(1 - \frac{1}{9}\right)^{-1}$

$= \left(\frac{8}{9}\right)^{-1} = \frac{9}{8}$

h) $(32 + 4^{-3})^0$ $= \underbrace{\left(32 + \frac{1}{4^3}\right)}_{\neq 0}^{0} = 1$

i) $\left(\frac{5^2 x^3 y^{-3}}{125\, xy}\right)^{-1}$ $= \frac{\cancel{125}^{5}\, xy}{\cancel{25}\, x^3 y^{-3}} = \frac{5\cancel{x}yy^3}{\cancel{x^3}_{x^2}} = \frac{5y^4}{x^2}$

or $5x^{-2}y^4$

j) $[(2x^{-3}y^{-2})^2]^{-2}$ $= \left(\left(\frac{2}{x^3y^2}\right)^2\right)^{-2} = \left(\frac{2}{x^3y^2}\right)^{-4}$

$= \left(\frac{x^3y^2}{2}\right)^4 = \frac{x^{12}y^8}{2^4}$

$= \frac{x^{12}y^8}{16}$

k) $\frac{u^{-1} - v^{-1}}{u^{-1} + v^{-1}}$ $= \left(\frac{\frac{1}{u} - \frac{1}{v}}{\frac{1}{u} + \frac{1}{v}}\right) \frac{\frac{uv}{1}}{\frac{uv}{1}}$

$= \frac{\frac{\cancel{u}v}{\cancel{u}} - \frac{u\cancel{v}}{\cancel{v}}}{\frac{\cancel{u}v}{\cancel{u}} + \frac{u\cancel{v}}{\cancel{v}}} = \frac{v-u}{v+u}$

Scientific Notation

$\underline{\ a\ } \times \underline{10^{b}}$

b power on 10

a # between 1.0 and 9. something

$1 \leq a < 10$

Put into scientific notation:

a) .000000000328

$3.28 \times 10^{-10}$

b) 1,248,000,000

$1.248 \times 10^{9}$

Put in standard notation:

a) $3.1 \times 10^{8}$

31,0000000

1 2 3 4 5 6 7 8

310,000,000

b) $2.3 \times 10^{-5}$

0.000023