MATH 1010 ~ Intermediate Algebra

Chapter 3: GRAPHS AND FUNCTIONS

Section 3.6: Relations and Functions

Objectives:

- Identify the domain and range of a relation.
- Determine if a relation is a function by inspection.
- Use function notation and evaluate functions.
- Identify the domain and range of a function.

A relation is a set of ordered pairs: (like a relationship)

inputs: 2, 1, 8, 5

a) $\{ (2,3), (1,5), (8,4), (5,3) \}$ outputs: 3, 5, 4

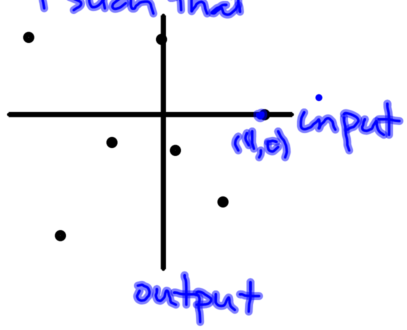
b) The set of first names paired with last names in a large class

{ (Joe, Anderson), (Mary, Smith), (Vince, Lowell) }

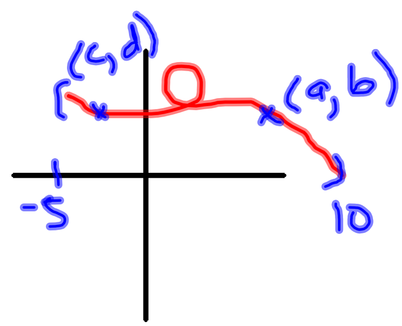
c) $\{ (s,N) | \ s = \text{social security number}, N = \text{name}\}$

such that

d)



e)



Vocab

Domain

Set of allowable inputs

Range set of outputs

A *function*, f from set A to set B, is a rule of correspondence that assigns to each element of the domain, x, exactly one element, y, in set B.

f is name of function

fn: each input has exactly one output

function

a) { (2,3), (1,5), (8,4), (5,3) }

yes it's a fn

b) The set of first names paired with last names in a large class

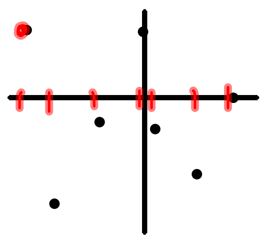
yes { (Andrew, Daniels), (Chris, Daniels) }

no { (Andrew, Daniels), (Andrew, Cummings) }

c) { (s,N)| s = social security number, N = name}

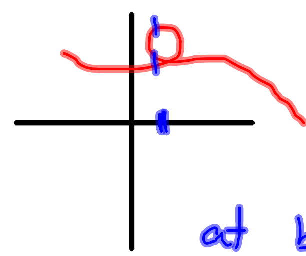
yes

d)



yes

e)



at blue mark x-value has 2 or 3 y-values on graph

⇒ not a function

① EXAMPLE:

Do these relationships describe a function?

a) Input: student in this class
Output: final grade in the class

yes, every student gets only one grade

b) Input: State
Output: number of senators from that state.

(AZ, 2), (UT, 2)

yes

c) Input: Adults who drive cars
Output: Cars they drive

because an adult drives more than one car, this is NOT a fn.

d) Input: Friend's name
Output: Friend's phone number

Della has several phone #s

$\Rightarrow$ NOT fn

Vocabulary:

f(x) function notation

read as "f of x" ; f is a fn of x;

x is input & f(x) is output

Independent variable

input (horiz. axis)

for f(x), x is indep. var

Dependent variable

output (vert. axis)

f(x)

f(a) means output for f(x) when we plug a in for x; f evaluated at a

f(2) means

output of f(x) when x=2

② EXAMPLE:

Evaluate this function at the given x-values:

$$f(x) = \frac{x^2-6}{x+1}$$ defn of f(x)

a) $f(2) = \frac{2^2-6}{2+1} = \frac{4-6}{3} = \frac{-2}{3}$ pt $(2, \frac{-2}{3})$

b) $f(-3) = \frac{(-3)^2-6}{-3+1} = \frac{9-6}{-3+1} = \frac{3}{-2}$ pt $(-3, \frac{-3}{2})$

c) $f(\bigstar) = \frac{\bigstar^2-6}{\bigstar+1}$

d) $f(2) - f(1) = \left(\frac{2^2-6}{2+1}\right) - \left(\frac{1^2-6}{1+1}\right) = \frac{4-6}{3} - \frac{-5}{2}$

$= \frac{-2}{3} + \frac{5}{2}$

$= \frac{-4}{6} + \frac{15}{6} = \frac{11}{6}$

e) $f(t-1) = \frac{(t-1)^2-6}{(t-1)+1}$

$= \frac{(t-1)^2-6}{t}$

③ EXAMPLE:

Evaluate this piece-wise function for the given values.

fn that's defined in pieces

$$f(x) = \begin{cases} x^2-1 & \text{if } x \le 1 \\ 2x+1 & \text{if } x > 1 \end{cases}$$

a) $f(1) = 1^2 - 1 = 0$ (top) $(1, 0)$

b) $f(-2) = (-2)^2 - 1 = 4 - 1 = 3$ (top) $(-2, 3)$

c) $f(3) = 2(3) + 1 = 6 + 1 = 7$ (bottom) $(3, 7)$

④ EXAMPLE:

f(x) = 3x - 7

find f(x+h) - f(x)

$$\underbrace{f(x+h)} - \underbrace{f(x)} = \overbrace{\left(3(x+h) - 7\right)}^{f(x+h)} - \overbrace{\left(3x - 7\right)}^{f(x)}$$

$$= \cancel{3x} + 3h - \cancel{7} - \cancel{3x} + \cancel{7}$$

$$= 3h$$

⑤ EXAMPLE: For each of these functions write the domain.

potential problems

① can't divide by zero

② can't take even root of neg. #

a) r(x) = { (2,1), (3,2), (1,5), (4,1) }

inputs: (domain) {2, 3, 1, 4}

range: {1, 2, 5}

b) $f(x) = \sqrt{x+1}$

can't take sqrt of negative #

domain: $x+1 \geq 0$ $\boxed{x \geq -1}$

c) $g(x) = \frac{2x-1}{3x+2}$

$3x+2 \neq 0$

domain: $x \neq \frac{-2}{3}, x \in \mathbb{R}$

d) $k(x) = x^2-3x+2$

domain: $x \in \mathbb{R}$ or $(-\infty, \infty)$

e) $g(x) = \frac{1}{(2x+1)(x-2)}$

$2x+1 \neq 0$ $x-2 \neq 0$

$x \neq \frac{-1}{2}$ $x \neq 2$

domain: $x \in \mathbb{R}, x \neq \frac{-1}{2}, 2$