Name: ______________________________

School: ______________________________

Grade: ______________________________

2020 State Math Contest (Junior Exam)

Weber State University

March 4, 2020

Instructions:

- Do not turn this page until your proctor tells you.
- Enter your name, grade, and school information following the instructions given by your proctor.
- Calculators are not allowed on this exam.
- This is a multiple-choice test with 40 questions. Each question is followed by answers marked (a), (b), (c), (d), and (e). Only one answer is correct.
- Mark your answer to each problem on the bubble sheet Answer Form with a #2 pencil. Erase errors and stray marks. Only answers properly marked on the bubble sheet will be graded.
- Scoring: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
- You will have 2 hours and 30 minutes to finish the test.
- You may not leave the room until at least 10:30 a.m.

---

1. You made a scale model of Earth and the moon. The Earth's diameter is about 1.3×10^4 kilometers. In the model the Earth's diameter is 50 centimeters. The moon's diameter is about 3.5×10^3 kilometers. Find the diameter of the moon in your model.

   (a) 12 centimeters
   (b) 13 centimeters
   (c) 14 centimeters
   (d) 15 centimeters
   (e) 16 centimeters

2. What is the sum of all solutions (real, complex, and repeated) for the following polynomial equation: $x^5 - 4x^3 + 5x^2 + x - 7 = 0$.

   (a) 1
   (b) 0
   (c) -4
   (d) -7
   (e) None of the above

3. How many squares are there in the infinite set $\{4, 44, 444, 4444, 44444, ...\}$?

   (a) One square
   (b) Two squares
   (c) Five squares
   (d) Eleven squares
   (e) Infinitely many squares

4. An air traffic control radar screen is a circle with a diameter of 24 inches. The radar screen is set to have a scale of 6 inches : 25 nautical miles. What is the area of the circular region covered by the radar?

   (a) 157 nautical miles^2
   (b) 314 nautical miles^2
   (c) 7,850 nautical miles^2
   (d) 31,400 nautical miles^2
   (e) None of the above.

5. Using a standard deck of 52 cards, how many ways are there to form a 5-card hand that has 3 cards of one rank and 2 cards of another rank.

   (a) 156
   (b) 468
   (c) 1,872
   (d) 3,744
   (e) 4,212

6. Brady is tinkering with new chocolate milk recipes. He has made a 10% chocolate milk solution (this means the solution is 10% chocolate and 90% milk). He also has 30 gallons of a 25% chocolate milk solution. The 10% solution isn't very tasty, and the 25% solution is too chocolaty. Brady is convinced that a 15% solution will be perfect. Brady plans to add 10% solution to the 25% until it is a 15% solution. Assuming he has plenty of 10% solution, how many gallons of 15% solution can he make?

   (a) 10 gallons
   (b) 20 gallons
   (c) 40 gallons
   (d) 60 gallons
   (e) 90 gallons

7. A ball is released from a height of 10 feet. Each time the ball hits the ground it will bounce directly back up $\frac{3}{4}$ of the distance it has fallen. What is the total distance the ball travels before it stops bouncing?

   (a) 70 feet
   (b) 80 feet
   (c) 60 feet
   (d) 75 feet
   (e) None of the above

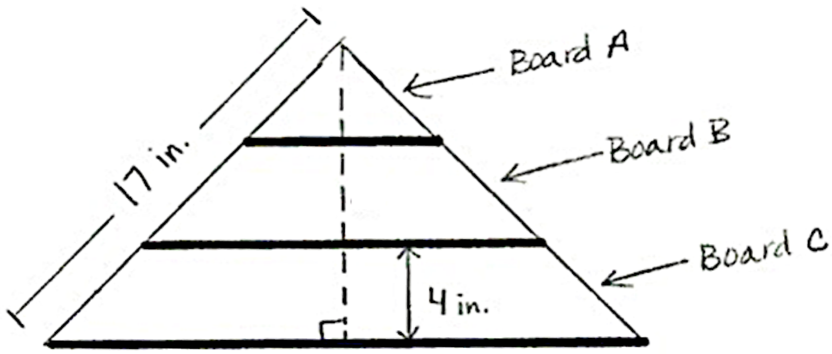
8. The function $f(x) = -\frac{2x}{x-1}$ is one-to-one. Its inverse is

   (a) $f^{-1}(x) = \frac{x}{x+2}$
   (b) $f^{-1}(x) = \frac{x-1}{-2x}$
   (c) $f^{-1}(x) = \frac{-2x}{y} + 1$
   (d) $f^{-1}(x) = \frac{x}{2(x+1)}$
   (e) $f^{-1}(x) = \frac{-2y+x}{x}$

9. A palindrome is a number that remains the same even if you reverse the order of its digits. For example, the number 15,351 is a palindrome. What is the largest positive factor of every four-digit palindrome?

   (a) 1
   (b) 2
   (c) 7
   (d) 9
   (e) 11

10. Jason wants to build a corner shelf for his living room. Rather than building the shelf out of a solid piece of wood, he plans to construct it out of three 4-inch wide boards, as shown in the diagram below.



If he wants the total length of the shelf along the wall sides to be 17 inches, what will the length of the **front** edge of the shelf (indicated by the **bold** line on board C) be? Round to the nearest inch.

(a) 17 inches
(b) 24 inches
(c) 578 inches
(d) 12 inches
(e) 20 inches

11. Using the diagram in the previous problem (#10), what is the length of the **front** edge of boards B and A (indicated by the **bold** lines on each board), in that order? Round to the nearest inch.

(a) 8 inches, 4 inches
(b) 9 inches, 5 inches
(c) 13 inches, 7 inches
(d) 16 inches, 8 inches
(e) Not enough information to compute.

12. A committee is to be formed consisting of 4 women and 3 men. There are 12 women and 8 men available for the committee to be formed. How many possible committees are possible?

(a) 495
(b) 56
(c) 390,700,800
(d) 27,720
(e) 77,520

13. A trader purchased a bracelet at a certain price. The trader then marked up the price 70% and attempted to sell it at that price. The bracelet sat in the case for days before the trader cut the sale price by 25%. The next day someone bought the bracelet. What percent profit did the trader make?

    (a) 2.8%
    (b) 17.5%
    (c) 27.5%
    (d) 35.7%
    (e) 45%

14. List all the possible values of $\frac{x}{|x|} + \frac{y}{|y|} + \frac{xy}{|xy|}$, where $x, y \in \mathbb{R}$.

    (a) $\{-3, 3\}$
    (b) $\{-3, 0, 3\}$
    (c) $\{-3, -1, 1, 3\}$
    (d) All real numbers
    (e) None of above

15. Find the units digit of 3^{999}.

    (a) 1
    (b) 3
    (c) 5
    (d) 7
    (e) 9

16. Washington School sent their 4 best chess players to challenge the 4 best chess players from Lincoln School. Each of the 4 Washington players will be randomly paired with a different Lincoln player for the first round of games. How many different possible pairings are possible for the first round of games?

    (a) 4
    (b) 16
    (c) 24
    (d) 576
    (e) 40,320

17. For all real numbers b and c such that the product of c and 3 is b, which of the following expressions represent the sum of c and 3 in terms of b?

    (a) $b + 3$
    (b) $3(b + 3)$
    (c) $3b + 3$
    (d) $\frac{b}{3} + 3$
    (e) $\frac{b+3}{3}$

18. If $a < 0$, then the quadratic function $y = x^2 + ax + a$ has

    (a) One zero
    (b) Two real zeros
    (c) None
    (d) Two conjugate complex zeros
    (e) None of above

19. What is the largest possible value for the sum of two fractions such that each of the four 1-digit prime numbers occurs as one of the numerators or denominators?

(a) $\frac{3}{2} + \frac{7}{5}$
(b) $\frac{5}{7} + \frac{2}{3}$
(c) $\frac{5}{2} + \frac{7}{3}$
(d) $\frac{7}{2} + \frac{5}{3}$
(e) $\frac{9}{5} + \frac{7}{3}$

20. The red part of Santa's hat is made from material that forms a cone. If the cone has a base diameter of 10 inches and a height of 12 inches, how much material is needed to make the red part of Santa's hat?

(a) 78.5 inches^2
(b) 188 inches^2
(c) 204 inches^2
(d) 314 inches^2
(e) 408 inches^2

21. The point $P = (-5, 5)$ is reflected over the line $y = 3x$, resulting in a point Q. What is the x-coordinate of point Q?

(a) 4
(b) 5
(c) 6
(d) 7
(e) 8

22. Two teachers and three students sit randomly around a round table. What is the probability each student sits next to at least one teacher?

   (a) 50%
   (b) 60%
   (c) 40%
   (d) 100%

23. A ladder is leaning against a vertical wall with its bottom 15 feet from the wall. If the bottom of the ladder is pulled out 9 feet farther away from the wall, its upper end slides 13 feet down. What is the length of the ladder?

   (a) 25 feet
   (b) 8 yards
   (c) 30 feet
   (d) 20 feet
   (e) 7 yards

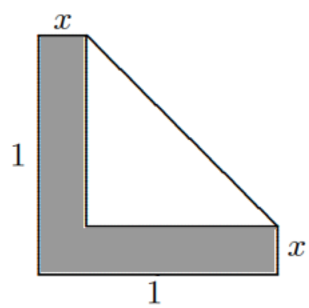
24. If $(1,2) \in A \cap B$, $A = \{(x,y)|ax+by=1\}$, $B = \{(x,y)|2ax+by^2=2\}$, then $a+b=$

   (a) 3
   (b) 1
   (c) -1
   (d) All real numbers
   (e) None of above

25. Note that $5625 = 3^2 \cdot 5^4$. How many positive integer divisors does 5625 have?

(a) 6
(b) 7
(c) 15
(d) 16
(e) None of the above.

26. In the figure below, if the area of the letter L part equals the area of the triangle, and the length of the base and the height is 1 unit, what is the length x of the ends of the letter L?



(a) $\sqrt{\frac{1}{3}}$ units
(b) $\sqrt{\frac{2}{3}}$ units
(c) $1 - \sqrt{\frac{1}{3}}$ units
(d) $1 - \sqrt{\frac{2}{3}}$ units
(e) $1 + \sqrt{\frac{2}{3}}$ units

27. It is known that in a certain population 10% of the population is afflicted by disease A, 15% are afflicted by disease B, and 5% are afflicted by both diseases. What proportion of the population is afflicted by disease B if they are known to have disease A?

(a) 50%
(b) 20%
(c) 5%
(d) 33.33%
(e) 1.5%

28. Find the exact value of $\sqrt{3+2\sqrt{2}} - \sqrt{3-2\sqrt{2}}$.

(a) 2
(b) 1
(c) -2
(d) 3
(e) None of the above

29. A temperature of 0° Celsius is equivalent to 32° Fahrenheit. At what temperature, if any, will a Fahrenheit thermometer and Celsius thermometer have the same reading? Assume that water boils at a temperature of 100°C (212°F).

(a) $-40°$
(b) $-10°$
(c) 20°
(d) 40°
(e) There is no temperature when the two scales produce the same reading.

30. A rhombus has sides of length 10 units, and its diagonals differ by 4 units. What is its area?

(a) 12 units^2
(b) 24 units^2
(c) 48 units^2
(d) 96 units^2
(e) 192 units^2

31. In a group of five friends, the sum of the ages of each group of four of them are 89, 90, 92, 94, and 95. What is the age of the youngest?

    (a) 18
    (b) 20
    (c) 21
    (d) 24
    (e) None of the above

32. John wants to buy a backpack but he is $18 short. For the same backpack, Kate is $7 short, Nancy is $6 short, and Bubba is $4 short. Which two of them, combining their money, can **certainly** buy the backpack?

    (a) John and Kate
    (b) John and Nancy
    (c) Nancy and Bubba
    (d) All the above.
    (e) None of the above

33. In the sequence of numbers $1, 4, 3, -1, ...$ each term after the first two is equal to the term preceding it minus the term preceding that. Find the sum of the first one hundred terms of the sequence.

    (a) 1
    (b) -2
    (c) -3
    (d) 7
    (e) 3

34. Square $ABCD$ has sides of length 3 units. Side AB is extended through B to E with $BE = 1$ unit. Segment DE intersects side BC at point F. What is the area of the triangle CDF?

   (a) $\frac{27}{8}$ units^2
   (b) 3 units^2
   (c) 4 units^2
   (d) $\frac{25}{6}$ units^2
   (e) 6 units^2

35. Each box of cereal contains a toy. The toy comes in 4 different colors: red, orange, green, or blue. Each of the 4 colors is equally likely to occur. Alex wants to get a toy in each of the 4 colors. If Alex buys 4 boxes of cereal, what is the probability that they get one toy of each of the 4 colors?

   (a) $\frac{1}{256}$
   (b) $\frac{1}{35}$
   (c) $\frac{3}{32}$
   (d) $\frac{1}{4}$
   (e) $\frac{3}{4}$

36. A goat, a horse, and a cow mistakenly enter a farmer's wheat field and eat some stalks of wheat. The horse eats twice as many stalks as the goat, and the cow eats twice as many stalks as the horse. The farmer demands 5 tou of wheat from the owners of the animals to replace what was eaten. How much wheat should be replaced by the horse's owner?

   (a) $\frac{5}{7}$ tou
   (b) 1 tou
   (c) $\frac{10}{7}$ tou
   (d) 2 tou
   (e) $\frac{20}{7}$ tou

37. There are three, three-digit positive integer numbers. None of them contains the digit 0, but all the remaining nine digits 1, 2 ,..., 9, appear exactly once in each number. Suppose that, from each given number, a new number is formed where the first and the last digits are exchanged. For example, if the three given numbers were 267, 813, 594, then the three new numbers would be 762, 318, 495. Now, assume that the sum of the three given numbers is 1665. What will be the sum of the three new numbers with the first and third digits exchanged?

    (a) 5661
    (b) 4995
    (c) 1665
    (d) 999
    (e) None of above

38. A positive whole number leaves a remainder of 7 when divided by 11 and a remainder of 10 when divided by 12. What is the remainder when divided by 66?

    (a) 0
    (b) 40
    (c) 28
    (d) 52
    (e) 29

39. Consider the square $ABCD$ and extend side AB through B to point E. Segment DE intersects side BC at point F. If the ratio of area of the triangle EBF to the area of the triangle CDF is $\frac{1}{9}$, what is the ratio of the length of segment BE to the length of side AB?

    (a) $\frac{1}{3}$
    (b) $\frac{1}{2}$
    (c) $\frac{3}{2}$
    (d) $\frac{2}{3}$

40. A 1-meter measuring stick is cut at a randomly selected location, creating two sticks. What is the probability that the larger of the two sticks is over twice as long as the shorter of the two sticks?

    (a) $\frac{1}{4}$
    (b) $\frac{1}{3}$
    (c) $\frac{1}{2}$
    (d) $\frac{2}{3}$
    (e) $\frac{3}{4}$