Name: ____________________

Student ID: ____________________

State Math Contest (Senior)

Instructions:

- Do not turn this page until your proctor tells you.
- Enter your name, grade, and school information following the instructions given by your proctor.
- Calculators are **not** allowed on this exam.
- This is a multiple choice test with 40 questions. Each question is followed by answers marked a), b), c), d), and e). Only one answer is correct.
- Mark your answer to each problem on the bubble sheet Answer Form with a #2 pencil. Erase errors and stray marks. Only answers properly marked on the bubble sheet will be graded.
- **Scoring:** You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
- You will have 2 hours and 30 minutes to finish the test.

1. Which of the following could not be the lengths of the sides of a triangle?

a) 5, 7, 8
b) 2, 4, 5
c) 4, 7, 10
d) 1, 1, 1
e) 3, 6, 10

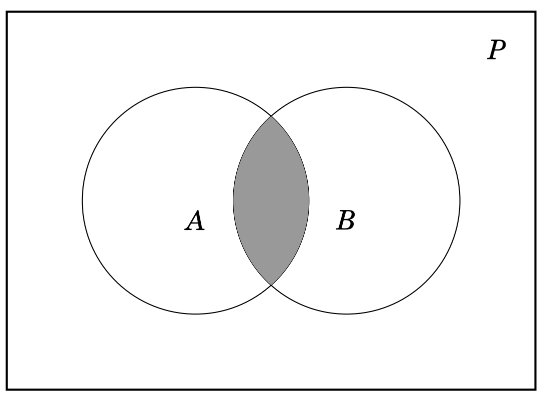
2. A bumble bee is traveling back and forth between the front end of two trains moving towards each other. If the trains start 90 miles away from each other and one train is going 10 miles per hour while the other train is going 20 miles an hour and the bumble bee is traveling 100 miles per hour, how many miles does the bumble bee travel before being smashed by the two trains colliding?

a) 450 miles
b) 90 miles
c) 300 miles
d) 180 miles
e) 270 miles

3. Four people's car keys are accidentally dropped into a pond. When the keys are fished out, they look alike except for the notches, so the four keys are returned in *random* order to the four owners. What is the probability that none of the owners will be able to get into his or her own car later?

a) 1/4
b) 3/8
c) 5/12
d) 1/2
e) 5/8

4. Which of the following expressions indicate the shaded region. ($\overline{A}$ is the complement of A relative to P.)



I. $A \cup B$ II. $A \cap B$ III. $P \cup (A \cup B)$ IV. $P \cap (A \cup B)$
V. $P \cap (A \cap B)$ VI. $P \cup (A \cap B)$ VII. $\overline{\overline{A} \cup \overline{B}}$ VIII. $\overline{\overline{A} \cap \overline{B}}$

a) only II.
b) only V.
c) only VIII.
d) II., V., and VII.
e) III., V., and VIII.

5. A rectangle is partitioned into 4 subrectangles as shown below. If the subrectangles have the indicated areas, find the area of the unknown rectangle.

60	100
?	80

a) 72 b) 21 c) 70

d) 48 e) 64

6. In the expansion of $(2x+y)^{30}$, what is the coefficient of x^2y^{28}?

a) 312 b) 435 c) 392

d) 1568 e) 1740

7. If $\cos(\theta)+\sin(\theta)=\frac{\sqrt{10}}{5}$ and $\tan(\theta)=\frac{a}{b}$, then $\frac{a}{b}+\frac{b}{a}=$

a) $\frac{5}{3}$ b) $\frac{-10}{3}$ c) $\frac{10}{3}$

d) $\frac{3}{10}$ e) 1

8. Two straight roads intersect at right angles. A car and bicycle meet at the intersection each traveling on a different road. The car is going 20 miles an hour and the bicycle is traveling at one fifth the speed of the car. After 30 minutes how far apart are they?

a) 10 miles b) $\sqrt{110}$ miles c) $\sqrt{104}$ miles

d) 11 miles e) 8 miles

9. Find the sum of all the even integers from 546 to 854 inclusive:

$$546+548+550+\cdots+852+854.$$

a) 106,400 b) 109,200 c) 107,800

d) 107,100 e) 108,500

10. Find the *shortest* path which starts at the origin and visits all five of the following points and returns to the origin: $\{(0,0), (1,0.5), (2,1), (2,0), (0,3)\}$.

a) $(0,0),(1,0.5),(2,1),(2,0),(0,3),(0,0)$ b) $(0,0),(2,0),(1,0.5),(2,1),(0,3),(0,0)$

c) $(0,0),(1,0.5),(2,0),(2,1),(0,3),(0,0)$ d) $(0,0),(0,3),(1,0.5),(2,0),(2,1),(0,0)$

e) $(0,0),(0,3),(2,0),(1,0.5),(2,1),(0,0)$

11. A ladder 20 feet long leans against a building. If the bottom of the ladder slides away from the building horizontally at a rate of 4 ft/sec, how fast is the ladder sliding down the house when the top of the ladder is 16 feet from the ground.

a) 3 ft/sec b) 4 ft/sec c) 2 ft/sec

d) 4 in/sec e) 5 ft/sec

12. Let z be a complex number of magnitude 1 such that z is not -1 or 1. Let $i = \sqrt{-1}$, and let c and d be real numbers with $c + di = \frac{1}{1+z}$. Which of the following is a possible value for c?

a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{3}}$ c) 1

d) $\frac{-1}{2}$ e) $\frac{-1}{\sqrt{3}}$

13. The number of real solutions to $\frac{x}{100} = \sin(x)$ is which of the following?

a) 32 b) 56 c) 63

d) 64 e) 65

14. Convert 6023.22 from base 7 to a decimal (base 10). What is the digit in the tens position?

a) 0 b) 1 c) 3

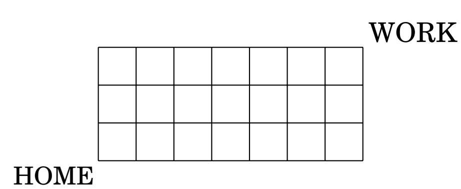
d) 5 e) 7

15. Let n be greatest integer that will divide the three integers 13511, 13903 and 14589 and leave the same remainder. Which is the sum of the digits of n?

a) 7 b) 9 c) 13

d) 17 e) 24

16. A young man has to walk from his home to his work. (His mom picks him up so he does not walk home.) How many days can he walk to work in a different way by walking a total of three blocks north and seven blocks east in any order? (No cutting diagonal through the blocks).



a) 1 b) 10 c) 80

d) 120 e) 100

17. Find the largest integer n such that n has exactly 4 positive divisors and n divides $100!$.

a) 716539 b) 8633 c) 5293

d) 29791 e) 100

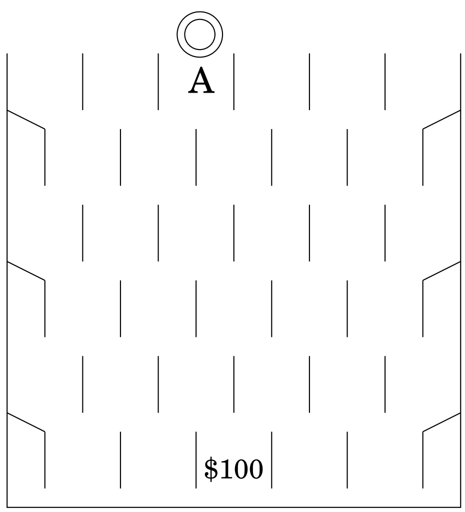
18. Consider the checkerboard below, with some of the squares removed as signified by the X's. How many ways are there to place 5 circles on the remaining squares (those without X's) so that no two circles are on the same row or column?

				X
			X	
		X		
	X			
X				

a) 44 b) 120 c) 40

d) 96 e) 45

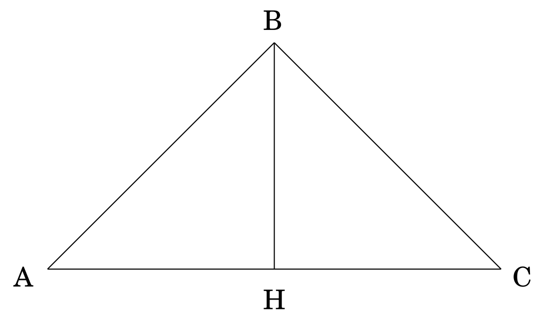
19. Elizabeth is a contestant on the *Price is Right* and her game is **PLINKO**. She has one token to drop from the top. On the game board below Elizabeth chooses slot **A** to drop her token into. At each divider there is an equal chance of going left and right. What is the probability Elizabeth will get the prize of $100?



a) $\frac{1}{5}$ b) $\frac{3}{16}$ c) $\frac{8}{32}$

d) $\frac{10}{31}$ e) $\frac{5}{16}$

20. The line BH is perpendicular to line AC and H is on the line AC. The angle BAC and BCA both measure $\frac{\pi}{6}$. If BH is length 4, what is the length of AC?
Diagram may not be drawn to scale.



a) $8\sqrt{3}$

b) $4\frac{\sqrt{3}}{2}$

c) 16

d) 10

e) $4\frac{\sqrt{2}}{2}$

21. Let $f(x)$ be an odd function defined on $\mathbb{R}$ satisfying

(a) $f(x+2) = -f(x)$, for all real numbers x;
(b) $f(x) = 2x$ when $0 \le x \le 1$.

Find the value of $f(10\sqrt{3})$.

a) $20\sqrt{3} - 36$

b) $-20\sqrt{3} + 36$

c) $20\sqrt{3} - 32$

d) $10\sqrt{3} + 18$

e) $-20\sqrt{3} - 36$

22. How many four-digit integers either leave a remainder of 2 when divided by 7, or a remainder of 4 when divided by 5, but not both?

a) 2570

b) 2572

c) 2575

d) 3084

e) 3086

23. Let a_n and b_n be sequences of numbers satisfying

$$a_1 = -1,\ b_1 = 2,\ a_{n+1} = -b_n,\ b_{n+1} = 2a_n - 3b_n \text{ for all natural number } n.$$

Compute $b_{2017} + b_{2016}$.

a) 3×2^{2015}

b) 3×2^{2017}

c) 3×2^{2014}

d) 3×2^{2016}

e) 5×2^{2016}

24. Find all real numbers a such that $f(x) = x^2 + a|x-1|$ is an increasing function on the interval $[0, \infty)$.

a) $[-2, \infty)$

b) $[-2, 0]$

c) $(-\infty, 0]$

d) $[0, 2]$

e) $[-2, 2]$

25. Find the area of triangle ΔABC if $AB = AC = 50$ in and $BC = 60$ in.

a) 2000 square inches
b) 1500 square inches
c) 1000 square inches
d) 2400 square inches
e) 1200 square inches

26. A group of airplanes is based on a small island. The tank of each plane holds just enough fuel to take it halfway around the world. Any desired amount of fuel can be transferred from the tank of one plane to the tank of another while the planes are in flight. The only source of fuel is on the island. It is assumed that there is no time lost in refueling either in the air or on the ground. The planes have the same constant speed and rate of fuel consumption. What is the smallest number of planes that will ensure the flight of one plane around the world on a great circle and have all the planes return safely to their island base?

a) 3
b) 4
c) 6
d) 7
e) 9

27. Find the units digit of 13^{2017}.

a) 5
b) 7
c) 9
d) 1
e) 3

28. Let $f(x) = \ln\left(\frac{1+x}{1-x}\right)$. What is $f\left(\frac{3x+x^3}{1+3x^2}\right)$ in terms of $f(x)$?

a) $f(x)$
b) $-3f(x) - e$
c) $\frac{f(x)+1}{f(x)-1}$
d) $f(x)^2 + f(x)$
e) $3f(x)$

29. Amy likes either red or green clothes, but not both. She likes either turtleneck or V-neck sweaters, but not both. Amy never wears a sweater that is both the color and the type she likes, nor does she wear one that is neither the color nor the type she likes. Amy wears a red turtleneck. If you want to buy a sweater for Amy that she will wear, should you buy a red V-neck, a green V-neck, or a green turtleneck?

a) A red V-neck
b) A green V-neck
c) A green turtle neck
d) Amy won't wear any of those sweaters
e) Not enough information is given to answer the question

30. The parabolas $y = x^2$ and $y = -x^2 + 4x - \frac{5}{2}$ have two common tangent lines. The one with smaller slope intersects the x-axis somewhere between $x = 0$ and $x = \frac{3}{5}$. Where?

a) At $x = 0$
b) At $x = \frac{1}{5}$
c) At $x = \frac{1}{4}$
d) At $x = \frac{1}{3}$
e) At $x = \frac{1}{2}$

31. Let $\frac{113}{50} = a + \cfrac{1}{b + \cfrac{1}{c + \cfrac{1}{d + \cfrac{1}{e}}}}$, where a, b, c, d and e are integers. Find c and e.

a) $c = 1$ and $e = 2$
b) $c = 3$ and $e = 1$
c) $c = 2$ and $e = 3$
d) $c = 2$ and $e = 4$
e) $c = 3$ and $e = 5$

32. Find the greatest common divisor of 123432123432100 and 2468642468642000015.

a) 1
b) 3
c) 5
d) 7
e) 15

33. Consider the sequence $x_0 = 1$, $x_1 = 2$, $x_2 = \frac{5}{2}$, $x_3 = \frac{11}{4}$, $x_4 = \frac{23}{8}$, ... The pattern is $x_n = \frac{3x_{n-1} - x_{n-2}}{2}$ for $n > 1$. Find the limit of these numbers.

a) 2
b) 2.5
c) 2.9
d) 3
e) 3.4

34. Find the area of overlap between the closed disc $x^2 + y^2 \leq 2$ and the parabolic region $y \geq x^2$.

a) $\frac{\pi}{3} + \frac{\sqrt{2}}{2}$
b) 2.5.
c) $\frac{\pi}{6} + 3\sqrt{2}$
d) 2.
e) $\frac{\pi}{2} + \frac{1}{3}$

35. How many pairs of integers (x, y) satisfy the following equation.

$$x^3 + 6x^2 + 8x = 3y^2 + 9y + 1.$$

a) 0
b) 1
c) 2
d) 3
e) None of the above

36. Find the shortest distance from the point $(16, 1/2)$ to the parabola $y = x^2$ in the plane.

a) $\frac{7}{2}\sqrt{17}$ units
b) 14 units
c) 15 units
d) $10\sqrt{2}$ units
e) 10 units

37. $\sqrt[3]{2\sqrt{13}+5} - \sqrt[3]{2\sqrt{13}-5} =$

a) -1
b) $2\sqrt{13}$
c) 10
d) 1
e) 3

38. Let S be a subset of the set $\{1, 6, 16, 30, 57, 113, 233, 465, 931, 1856, 3717, 7432, 14865, 29731, 59454\}$ so that the elements in S add up 35573. Find the two smallest elements of S.

a) 1 and 3717
b) 233 and 1856
c) 113 and 233
d) 6 and 931
e) 6 and 30

39. Suppose real numbers x, y, z satisfy the equation $\frac{x}{y+z} + \frac{y}{z+x} + \frac{z}{x+y} = 1$.

Compute the value of

$$\frac{x^2}{y+z} + \frac{y^2}{z+x} + \frac{z^2}{x+y}.$$

a) 1
b) -1
c) 2
d) 0
e) -2

40. Let $f(x) = 4\sin^3 x - \sin x + 2\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right)^2$. What is the minimal period of $f(x)$?

a) 2π
b) $\pi/2$
c) $2\pi/3$
d) $\pi/3$
e) π

END OF EXAM