Solutions for Test 4

1. Since \( n = 64 > 40 \), we can use the large sample procedure and the confidence interval at \( \alpha = 0.05 \) is

\[
511 \pm 1.96 \times \frac{60}{\sqrt{64}}
\]

or \((496.3, 525.7)\). This calculation does not need the assumption of normal distribution since \( n \) is large enough.

If the sample size is 20, then it won’t allow us to use the large sample procedure, and we won’t be able to proceed with the calculation since we don’t have any information about the population distribution.

2. (a) The critical \( t \) value is \( t_{0.05,4} = 2.132 \). The 95% lower confidence bound is

\[
\mu \leq 500 + 2.132 \times \frac{75}{\sqrt{5}} = 571.5
\]

(b) Here we need to use \( t_{\alpha/2,4} = 2.776 \). The prediction interval is

\[
500 \pm 2.776 \times 75 \times \sqrt{\frac{1 + \frac{1}{5}}{1}}
\]

or \((271.9, 728)\). We can interpret this as saying that with probability 0.95, next year’s snowfall would be between 271.9 and 728 inches.

3. (a) Since we know \( \sigma \) and the population is assumed to have a normal distribution, we can use the \( z \)-test. As

\[
z = \frac{73 - 75}{9/\sqrt{25}} = -1.11 > -z_{0.025} = -1.96,
\]

we won’t be able to reject \( H_0 \).

If instead, you only have \( s = 9 \) as the sample standard deviation, you would have to compare with \( t_{0.025,24} = 2.064 \) and the conclusion would be the same.

(b) If you double the sample size to 50, you could always use the large sample test.

\[
\beta(70) = 1 - \Phi \left( -1.96 + \frac{75 - 70}{9/\sqrt{50}} \right) = 1 - \Phi(1.968) = 1 - 0.9756 = 0.0244
\]

4. We first state \( H_0 : p = 0.2 \) and \( H_a : p < 0.2 \). When \( n = 20 \), we have to use the binomial distribution function to compute probabilities.

\[
P(X \leq 2, p = 0.2) = B(2; 20, 0.2) = 0.206
\]
When the poll is enlarged to 200, we can use the normal distribution approximation.
If 30 households are found subscribing to the paper, \( \hat{p} = 0.15 \),

\[
z = \frac{0.15 - 0.2}{\sqrt{0.2 \times 0.8/200}} = -1.768 < -z_{0.05} = -1.645.
\]

So \( H_0 \) is rejected.