Calculus III Section 04
Sample Midterm I

Warning: The actual exam will have material not covered on this sample exam. This exam was written to give you a sense of the kinds of problems that will be asked. Please review all material covered in the homeworks.

1. The equations below give a parametric representation of a curve.

\[ x = -\sin t \]
\[ y = 4 \cos t \quad 0 \leq t < 2\pi \]

a. Graph the curve, include arrows to show the curve’s orientation.
b. Obtain the Cartesian equation of the curve by eliminating \( t \).
c. Find the equation of the tangent to the curve at \( t = \pi / 4 \).

2. Consider two vectors \( \mathbf{a} \) and \( \mathbf{b} \).

\[ \mathbf{a} = 4\mathbf{i} - 2\mathbf{j} \]
\[ \mathbf{b} = \mathbf{i} + 3\mathbf{j} \]

a. Find the angle between vectors \( \mathbf{a} \) and \( \mathbf{b} \) and make a sketch.
b. Find a vector that has the opposite direction of \( \mathbf{a} \) and has unit length.

3. The position of a moving particle at time \( t \) is given by the position vector \( \mathbf{r}(t) \).

\[ \mathbf{r}(t) = t^2\mathbf{i} + e^{2t}\mathbf{j} \]

a. Find the velocity vector of the particle, \( \mathbf{v}(t) \).
b. Find the acceleration vector of the particle, \( \mathbf{a}(t) \).
c. If a second particle moves with velocity \( \mathbf{v}(t) = 2t^3\mathbf{i} + e^{-t}\mathbf{j} \) and \( \mathbf{r}(0) = \mathbf{i} - 2\mathbf{j} \), what is the equation for its position vector \( \mathbf{r}(t) \)?

4. Consider the following position vector:

\[ \mathbf{r}(t) = \frac{1}{4} t^4 \mathbf{i} + \frac{1}{2} t^2 \mathbf{j} \]

a. Find the unit tangent vector \( \mathbf{T}(t) \) at point \( t_1 = \frac{1}{2} \).
b. Find the curvature \( k(t) \) at point \( t_1 = \frac{1}{2} \).
c. Make a sketch of the curve and the unit tangent vector \( \mathbf{T}(t) \) at point \( t_1 = \frac{1}{2} \).

5. a. Sketch a graph of the plane described by the equation \( 3x - 2y + z = 6 \).
b. Find the equation of the sphere whose center is \((-3, 2, -2)\) and that is tangent to the \( yz \)-plane.