Review Problems from Chapter 17, Part II

1. Evaluate the following surface integral, where $S$ is the triangular region with vertices $(1, 0, 0), (0, 2, 0)$ and $(0, 0, 2)$.

$$\iint_{S} xy \, dS$$

Hint: the triangle $S$ lies on the plane $2x + y + z = 2$.

2. Evaluate the following surface integral, where $S$ is the surface

$$z = \frac{2}{3}(x^{3/2} + y^{3/2}), \text{ where } 0 \leq x \leq 1, 0 \leq y \leq 1.$$ 

$$\iint_{S} y \, dS$$

3. Find the surface area of the part of the paraboloid $x = y^2 + z^2$ that lies inside the cylinder $z^2 + y^2 = 9$.

4. Consider the cone described by $G$: $z = 2 - \frac{1}{2}\sqrt{x^2 + y^2}, 1 \leq z \leq 2$. Let $\mathbf{F}$ be the vector field $\mathbf{F} = (yx^2)i - (xy^2)j + k$. Evaluate the flux of the curl of the vector field $\mathbf{F}$ across $G$ in the direction of the outward unit normal $\mathbf{n}$:

$$\text{flux}(\nabla \times \mathbf{F}) = \iint_{G} (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, dS.$$ 

5. Calculate the flux of $\mathbf{F}$ across the surface $G$: $z = \sqrt{1 - y^2}, 0 \leq x \leq 5$. Let $\mathbf{F}$ be the vector field $\mathbf{F} = yi - xj + 2k$.

$$\text{flux} \mathbf{F} = \iint_{G} \mathbf{F} \cdot \mathbf{n} \, dS$$