Math 1050-4 Midterm 2, October 18, 2006

Solutions

Problem 1 (15 points). Let \( f(x) = 2x^2 + 8x + 10 \).

a) Find the roots of this polynomial.

Solution:

\[
x_{1,2} = \frac{-8 \pm \sqrt{64 - 80}}{4} = \frac{-8 \pm \sqrt{-16}}{4} = \frac{-8 \pm 4i}{4} = -2 \pm i.
\]

b) Write the polynomial as a product of linear factors.

Solution:

\[
f(x) = 2(x + 2 - i)(x + 2 + i)
\]

Problem 2 (20 points). Let \( f(x) = x^3 + 4x^2 + x - 6 \).

a) List all possibilities for rational roots (using the rational root test).

Solution:

\[
\pm 1, \pm 2, \pm 3, \pm 6.
\]

b) By checking the list, find at least one rational root.

Solutions:

\[
f(1) = 1^3 + 4 \cdot 1^2 + 1 - 6 = 0.
\]

So, 1 is a root.

c) Factor the polynomial (using long division) and find all roots.

Solution: By long division we find that

\[
\frac{x^3 + 4x^2 + x - 6}{x - 1} = x^2 + 5x + 6.
\]
Hence
\[ x^3 + 4x^2 + x - 6 = (x - 1)(x^2 + 5x + 6). \]
The roots of
\[ x^2 + 5x + 6 = 0 \]
are
\[ x_{1,2} = \frac{-5 \pm \sqrt{25 - 24}}{2} = \frac{-5 \pm 1}{2}. \]
It follows that the roots are \(-3\) and \(-2\). Hence, we have
\[ x^2 + 5x + 6 = (x + 3)(x + 2). \]
All roots of the polynomial are \(-3\), \(-2\) and 1.

\[ d) \text{ Write the polynomial as a product of linear factors.} \]

\[ \text{Solution:} \]
\[ x^3 + 4x^2 + x - 6 = (x + 3)(x + 2)(x - 1). \]

**Problem 3 (15 points).** Perform the operations and write the result in standard form \(a + bi\) with \(a\) and \(b\) real numbers.

\[ \frac{2i}{2 + i} + \frac{5}{2 - i} \]

**Solution:**
\[ \frac{2i}{2 + i} + \frac{5}{2 - i} = \frac{2i(2 - i) + 5(2 + i)}{(2 + i)(2 - i)} = \frac{4i + 2 + 10 + 5i}{4 + 1} = \frac{12 + 9i}{5} = \frac{12}{5} + \frac{9}{5}i. \]

**Problem 4 (20 points).** Let
\[ f(x) = \frac{2x^2 - 5x + 5}{x - 2} \]
a) Find vertical asymptotes (if any).

**Solution:** There is one vertical asymptote \(x = 2\).
b) Find horizontal asymptotes (if any).

**Solution:** None.

c) Find slant asymptotes (if any).

**Solution:**

By long division we get

\[
\frac{2x^2 - 5x + 5}{x - 2} = 2x - 1 + \frac{3}{x - 2}.
\]

Hence, the slanted asymptote is \( y = 2x - 1 \).

**Problem 5 (15 points).** Find the solutions of the equation

\[
\log_{10}(x^2 + 1) - \log_{10}(x - 2) = 1.
\]

**Solution:**

\[
\log_{10}(x^2 + 1) - \log_{10}(x - 2) = \log_{10} \frac{x^2 + 1}{x - 2} = 1 = \log_{10} 10.
\]

This implies that

\[
\frac{x^2 + 1}{x - 2} = 10.
\]

Hence, we have

\[
x^2 + 1 = 10x - 20
\]

and

\[
x^2 - 10x + 21 = 0.
\]

By solving this quadratic equation, we get

\[
x_{1,2} = \frac{10 \pm \sqrt{100 - 84}}{2} = \frac{10 \pm \sqrt{16}}{2} = \frac{10 \pm 4}{2}
\]

and the roots are 3 and 7. By inspection, we see that both of these numbers are the solutions of our equation.

**Problem 6 (15 points).** Solve the system

\[
\begin{align*}
x + 2y &= 4 \\
3x - 9y &= -3
\end{align*}
\]

of linear equations.
Solution: By multiplying the first equation with $-3$ and adding to the second equation we get the system

\[
\begin{align*}
x + 2y &= 4 \\
-15y &= -15
\end{align*}
\]

From the second equation we see that $y = 1$. By backsubstitution, we get $x + 2 = 4$ and $x = 2$. Therefore, the solution is $x = 2$ and $y = 1$. 