1-3. Let \( p(t) = \frac{t}{18} \) be the density function for the shelf life of a brand of banana which lasts up to 6 weeks. Time \( t \) is measured in weeks and \( 0 \leq t \leq 6 \).

1. (5 pts) What is the cumulative distribution function value \( P(2) \) that the shelf life of a brand of banana lasts up to 2 weeks?

\[
P(2) = \int_{-\infty}^{2} p(t) \, dt = \int_{0}^{2} \frac{t}{18} \, dt = \left[ \frac{t^2}{36} \right]_0^2 = \frac{4}{36} = \frac{1}{9} \approx 11.1\%
\]

2. (5 pts) Find the mean time of the shelf life of a brand of a banana.

Mean time \( = \int_{-\infty}^{\infty} x \cdot p(x) \, dx = \int_{0}^{6} x \cdot \frac{x}{18} \, dx = \int_{0}^{6} \frac{x^2}{18} \, dx \)

\[
= \left[ \frac{x^3}{54} \right]_0^6 = \frac{6^3}{54} - \frac{0^3}{54} = \frac{4}{36} \text{ weeks}
\]

So a banana can last average for 4 weeks.

3. (5 pts) Find the median time of the shelf life of a brand of a banana. (You might use \( \sqrt{2} \approx 1.4 \).)

Find \( T \) such that \( \int_{-\infty}^{T} p(x) \, dx = 0.5 \).

\[
\int_{0}^{T} \frac{x}{18} \, dx = \frac{1}{2} \iff \left[ \frac{x^2}{36} \right]_0^T = \frac{1}{2} \iff \frac{T^2}{36} - \frac{0^2}{36} = \frac{1}{2}
\]

\[
\iff \frac{T^2}{36} = \frac{1}{2} \iff T^2 = 18 \iff T = \sqrt{18} = \sqrt{9 \cdot 2} = 3 \cdot \sqrt{2} \approx 3 \cdot 1.4 = 4.2 \text{ weeks}
\]

So half bananas can last up to 4.2 weeks.