# 1. Find \( c \) in \((-2, 2)\) such that \( f'(c) = \frac{f(2) - f(-2)}{2 - (-2)} \). Here \( f'(c) = 2c + 2 \).

So solve \( 2c + 2 = \frac{(2^2) - ((-2)^2) + (2)}{2 - (-2)} \) for \( c \).

**\( c = -\frac{1}{2} \)** in \((-2, 2)\).

# 2. Find \( c \) in \((1, 3)\) such that \( f'(c) = \frac{f(3) - f(1)}{3 - 1} \). Here \( f'(c) = \frac{-1}{2} \).

So solve \( \frac{-1}{2} = \frac{(\frac{3}{2} + 1) - (\frac{1}{2} + 1)}{3 - 1} \) for \( c \).

**\( c = 3 \)** \( \Rightarrow \) \( c = \pm \sqrt{3} \).

But we need to choose \( c = \sqrt{3} \) only, cause it must be in \((1, 3)\).

# 3. Let \( f(x) = x - \cos x \). Then \( f(0) = 0 - \cos 0 = -1 < 0 \) and \( f(\frac{\pi}{2}) = \frac{\pi}{2} - \cos \frac{\pi}{2} = \frac{\pi}{2} > 0 \).

So \( f(0) < 0 < f(\frac{\pi}{2}) \). Then \( f \) by **Intermediate Value Theorem (IVT)**

\( f(x) = 0 \) has at least one solution \( c \), that is, \( f(c) = 0 \) in \((0, \frac{\pi}{2})\).

# 4. \( f'(x) = -\frac{2}{x^2} < 0 \) for all \( x \) in \((-2, 1)\).

So by theorem, \( f \) must be increasing on \((-2, 1)\).

# 5. Let \( h(x) = f(x) - g(x) \) on \([1, 5]\). Then \( h'(x) = f'(x) - g'(x) = 0 \) by assumption.

So \( h'(x) = 0 \). Then \( h(x) = C \) (constant) on \([1, 5]\).

We need to determine \( C \). Since \( h(2) = f(2) - g(2) = 4 - 3 = 1 \), \( h(3) = \sqrt{3} \) and \( h(4) = h(4) = -3 \).

# 6. Since \( f'(x) = 0 \) on \([1, 4]\), \( f(x) = C \) (constant) on \([1, 4]\).

Since \( f(2) = 4 \), \( f(2) = 4 \) on \([1, 4]\). \( \Rightarrow f(2) = 4 \) on \([1, 4]\).

# 7. (1) Choose \( 0 < k < 1 \). Then \( f(0) = -1 < 0 \) & \( f(1) = 4 + 2 \cdot 1 = 5 > 0 \).

\( \Rightarrow \) (let \( f(x) = 4x^2 + 2x - 1 \) \( \Rightarrow \) \( f(0) < 0 < f(1) \).

Then by **IVT**, there exists at least one \( c \) in \((0, 1)\) such that \( f(c) = 0 \). \( \Rightarrow \) \( c \) is a solution of \( 4x^2 + 2x - 1 = 0 \).

(2). We have shown that \( f(x) = 0 \) has at least one solution by \#(1).

Suppose there are \( c_1 \) & \( c_2 \) as solutions of \( f(x) = 0 \).

Then \( f(c_1) = 0 = f(c_2) \). Then by **Rolle's theorem** (or **MVT**), there exists \( c \) between \( c_1 \) and \( c_2 \) such that \( f'(c) = 0 \).

But \( f'(x) = 12x + 2 \) has no solution, that is, there is no \( x \) such that \( 12x + 2 = 0 \). So such \( c \) that \( f'(c) = 0 \) cannot exist.

This contradiction happens because we assume that there are two solutions.

So \( f(x) = 0 \) has only one solution!