#1. (1) \( f(x) = -x^4 + 3x^3 + 9x - 12 \) on \([-2, 4]\).

\[ f'(x) = -4x^3 + 9x^2 + 9 = -3(x^2 - 2x - 3) = -3(x - 3)(x + 1) = 0 \Rightarrow x = -1, 3. \]

\[ f''(x) = -6x + 6 = -6(x - 1) = 0 \Rightarrow x = 1. \]

\[ \begin{array}{|c|c|c|c|}
\hline
x & f'(x) & f''(x) & f(x) \\
\hline
-2 & - & + & \\
-1 & + & + & \\
0 & + & + & - \\
1 & - & - & \\
3 & + & + & 0 \\
4 & - & - & \\
\hline
\end{array} \]

- Inflection point
- Local max
- Local min

\( f(1) = -1 \) \quad & \text{Inflection point}

\( f(3) = 15 \) \quad & \text{Local max}

\( f(-1) = -17 \) \quad & \text{Local min}

\[ \begin{cases}
  f(2) = -10 \\
  f(4) = 0
\end{cases} \] \quad & \text{End points}

(a) Critical points are \((-1, -17)\) and \((3, 15)\).

(b) Inflection point is \((1, 15)\), since \( f' \) changes concavity at \( x = 1 \).

(c) \( f \) is increasing if \( f' > 0 \), that is, \(-1 < x < 1\).

\( f \) is decreasing if \( f' < 0 \), that is, \( -2 < x < -1 \) and \( 3 < x < 4 \).

(d) \( f \) has a local max 15 at \( x = 3 \) & a local min -17 at \( x = -1 \).

(e) \( f \) has the global max 15 at \( x = 3 \) & the global min -17 at \( x = -1 \).

(f)

\( f(x) = 25 \sin x - x \) on \([0, \pi/2]\).

\[ f'(x) = 25 \cos x - 1 = 0 \Rightarrow \cos x = \frac{1}{5} \Rightarrow \frac{7\pi}{2}, \quad f''(x) = -25 \sin x = 0 \Rightarrow \pi = 0. \]

\[ \begin{array}{|c|c|c|c|c|}
\hline
x & 0 & \frac{\pi}{2} & \frac{\pi}{3} & \frac{\pi}{3} + \frac{\pi}{2} \\
\hline
f'(x) & + & 0 & - & - \\
\hline
f''(x) & - & - & 0 & 0 \\
\hline
f(x) & 0 & \frac{\pi}{2} & \frac{\pi}{3} & \frac{\pi}{3} + \frac{\pi}{2} \\
\hline
\end{array} \]

- End points
- Local max

(a) Critical point is \((\pi/3, \sqrt{3} - \pi/3)\).

(b) No inflection point, since 0 is one of the end points.

(c) \( f \) is increasing if \( 0 \leq x < \pi/3 \) & \( f \) is decreasing if \( \pi/3 < x < \pi/2 \).

(d) \( f \) has a local max \( \sqrt{3} - \pi/3 \) at \( x = \pi/3 \) & no local min.

(e) \( f \) has the global max \( \sqrt{3} - \pi/3 \) at \( x = \pi/3 \) & the global min 0 at \( x = 0 \).

(f)
(3) \( f'(x) = (6x - 4)' = (6x^{1/2} - 4)' = 6 \cdot \frac{1}{2} x^{-1/2} = 3 \cdot \sqrt{x/2} \),
\[ f''(x) = -\frac{3}{2} \frac{1}{\sqrt{x^3}} \]
\( f' \) is undefined at \( x = 0 \), \( f'' \) is undefined at \( x = 0 \).
\( f(0) = 6 \cdot 0 - 4 = -4 \).
\( f(2) = 6 \cdot \sqrt{2} - 4 = 6 \cdot 2^{1/2} - 4 \).
\( f(4) = 6 \cdot \sqrt{4} - 4 = 6 \cdot 2 - 4 = 8 \).

(a) Critical point in \((0, -4)\).
(b) No inflection point, since it is always concave down.
(c) \( f \) is increasing if \( 0 < x < 4 \).
(d) \( f \) has no local min / no local max.
(e) \( f \) has the local min \(-4\) at \( x = 0 \) & global max \( 8 \) at \( x = 4 \).

(4) \( f'(x) = e^x (x^2 - 2x + 1) + e^x (x - 2) = e^x (x^2 - 2x + 1 + x - 2) = e^x (x^2 - 1) = e^x (x + 1)(x - 1) \).
\( \Rightarrow x = 1, -1 \).
\( f''(x) = e^x (x^2 + 1) + e^x (2x) = e^x (x^2 + 2x - 1) = e^x (x^2 + 2x - 1) \).
\[ f(-2) = e^{-2} (9) = \frac{e^9}{e^2} \]
\[ f(1) = e^1 (2) = 2e \]
\[ f(3) = e^3 \cdot 4 = 4e^3 \]
\( f \) is increasing if \(-1 < x < 1 \) & \( 1 < x < 3 \),
\( f \) is decreasing if \(-3 < x < -1 \).
\( f \) has a local max \( \frac{4}{e} \) at \( x = 1 \), local min \( 0 \) at \( x = 1 \).
\( f \) has the global max \( 4e^3 \) at \( x = 3 \), global min \( 0 \) at \( x = 1 \).

(5) \( f'(x) = \ln(x-1) + (x-1)(x-1) = \ln(x-1) + 1 = 0 \Rightarrow x - 1 = e^{-1} \Rightarrow x = 1 + \frac{1}{e} \)
\( f''(x) = \frac{1}{x-1} \Rightarrow x \) is undefined at \( x = 1 \).

(a) Critical point in \((1, e^{-1})\).
(b) Inflection point \( 1 + \frac{1}{e} \).
(c) \( f \) is increasing if \( x > 1 + \frac{1}{e} \) & \( f \) is decreasing if \( 1 < x < 1 + \frac{1}{e} \).
(d) \( f \) has no local max & a local min \( -\frac{1}{e} \) at \( x = 1 + \frac{1}{e} \).
\( f \) has the global min \( -\frac{1}{e} \) at \( x = 1 + \frac{1}{e} \),
\( f \) has no global max (since \( f(x) \to \infty \) as \( x \to \infty \)).
2. Let \((x, y)\) be a point on \(y^2 = 4x^3\).

Let \(f\) be the square of the distance between \((x, y)\) and \((2, 0)\).

Then \(f = (x-2)^2 + (y-0)^2 = (x-2)^2 + y^2 = (x-2)^2 + (4x^3)\).

\[ f(x) = (x-2)^2 + 4x^3 \Rightarrow f'(x) = 2(x-2) + 12x^2 = 0 \Rightarrow x = 0. \]

\[ f''(x) = 2 > 0. \]

So \(f\) has a local **min** at \(x = 0\). __Local & global min__

If \(x = 0\), then the square of the distance from \((2, 0)\) has the minimum. \(\sqrt{(0-2)^2 + 0^2} = \sqrt{4} = 2\).

A point on \(y^2 = 4x\) at \(x = 0\) is \((0, 0)\).

So the point \((0, 0)\) minimizes the distance from \((2, 0)\) and the minimum distance is 2.

3. \(f'(x) = 0 \Rightarrow x = 1, -2.\)

\[ f''(x) = -(x+2)^2 - (x-1) \cdot 2(x+1) = -(x+2)(x+1+2(x-1)) \]

\[ = -(x+2)(3x) \Rightarrow x = 0, -2. \]

\[ f\] has a local **max** at \(x = 1\) and no local **min**.

A rough graph of \(f\) will be...

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**Table:**

<table>
<thead>
<tr>
<th>(x)</th>
<th>(-2)</th>
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<th>1</th>
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<tbody>
<tr>
<td>(f'(x))</td>
<td>-</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td>(f''(x))</td>
<td>-</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td><strong>Features</strong></td>
<td>Inflection</td>
<td>Inflection</td>
<td>local max</td>
</tr>
</tbody>
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