**Linear Transformation**

A linear transformation is a function $T$ defined on a vector space $V$ with range in a vector space $W$ satisfying the rules

(a) $T(v_1 + v_2) = T(v_1) + T(v_2)$
(b) $T(kv_1) = kT(v_1)$.

**Theorem 1 (Matrix of $T$)**
Assume $V = \mathbb{R}^n$ and $W = \mathbb{R}^m$. Then $T$ is represented as a matrix multiply

$$T(x) = Ax$$

where $A$ is the $n \times m$ matrix whose columns are given in terms of the identity matrix $I$ and function $T$ by the formula

$$\text{col}(A, j) = T(\text{col}(I, j)), \quad j = 1, \ldots, n.$$ 

**Definition**: A basis of a vector space $V$ is a set of vectors $v_1, \ldots, v_n$ such that every vector $v$ in $V$ can be uniquely written as a linear combination of $v_1, \ldots, v_n$. Briefly, the vectors span $V$ and are independent.

**Theorem 2 (Representation of $T$)**
Every basis \{ $v_1, \ldots, v_n$ \} of $V$ gives a relation

$$T \left( \sum_{j=1}^{n} c_j v_j \right) = \sum_{j=1}^{n} c_j w_j, \quad \text{where} \quad w_j = T(v_j).$$