Definition

The Laplace Transform $F(s) =$

Laplace Transforms

Transform

$$\mathcal{L}\{1\}$$

Laplace Transforms

Transform

$$\mathcal{L}\{t\}$$

Laplace Transforms

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)dt.$$

Def. 7.1.1, p. 442.

$$\mathcal{L}\{1\} = \frac{1}{s}.$$

$$\mathcal{L}\{t\} = \frac{1}{s^2}.$$

Transform

$$\mathcal{L}\{t^n\}$$

Transform

$$\mathcal{L}\{\cos(kt)\}$$

Transform

$$\mathcal{L}\{\sin(kt)\}$$

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}.$$

$$\mathcal{L}\{\cos(kt)\} = \frac{s}{s^2 + k^2}.$$

$$\mathcal{L}\{\sin(kt)\} = \frac{k}{s^2 + k^2}.$$

Transform

$$\mathcal{L}\{\cosh{(kt)}\}$$

Laplace Transforms

Transform

$$\mathcal{L}\{\sinh{(kt)}\}$$

Laplace Transforms

S-transform

$$\mathcal{L}\{e^{at}f(t)\}$$

Laplace Transforms

$$\mathcal{L}\{\cosh(kt)\} = \frac{s}{s^2 - k^2}.$$

$$\mathcal{L}\{\sinh(kt)\} = \frac{k}{s^2 - k^2}.$$

$$\mathcal{L}\{e^{at} f(t)\} = F(s - a).$$

$$\mathcal{L}^{-1}\{F(s - a)\} = e^{at} f(t).$$

Def. 7.3.1, p. 465.

S-translation

$$\mathcal{L}\{e^{at}\}$$

Laplace Transforms

S-translation

$$\mathcal{L}\{e^{at}\cos(kt)\}$$

Laplace Transforms

S-translation

$$\mathcal{L}\{e^{at}\sin(kt)\}$$

Laplace Transforms

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}.$$

$$\mathcal{L}\{e^{at}\cos(kt)\} = \frac{s-a}{(s-a)^2+k^2}.$$

$$\mathcal{L}\{e^{at}\sin(kt)\} = \frac{k}{(s-a)^2+k^2}.$$

S-translation

$$\mathcal{L}\{\frac{e^{at}}{2k}t\sin(kt)\}$$

Laplace Transforms

S-translation

$$\mathcal{L}\{\frac{e^{at}}{2k^3}(\sin(kt) - kt\cos(kt)\}$$

Laplace Transforms

Definition

Exponential order

Laplace Transforms

$$\mathcal{L}\{\frac{e^{at}}{2k}t\sin(kt)\} = \frac{s-a}{((s-a)^2+k^2)^2}.$$

$$\mathcal{L}\{\frac{e^{at}}{2k^3}(\sin(kt) - kt\cos(kt)\} = \frac{1}{((s-a)^2+k^2)^2}.$$

$|f(t)| \leq Me^{ct}$ for $t \geq T$.

Note: $\frac{|f(t)|}{Me^{ct}} \leq 1$.

Show: For any M, c, that Me^{ct} grows faster than $|f(t)|$, *i.e.* - it converges, not diverges.

Eq. 7.1.23, p. 448.

Theorem

Existence of a Laplace Transform

Theorem

Transforms of Derivatives

Theorem

Transforms of Higher Derivatives

If the function f is piecewise continuous for $t \geq 0$, and it is of exponential order as $t \to \infty$, then its Laplace transform $\mathcal{L}\{f(t)\} = F(s)$ exists.

Thm. 7.1.2, p. 448.

If the function $f(t)$ is (1) piecewise continuous and smooth, and (2) it is of exponential order, then -

$$\mathcal{L}\{f'\} = sF(s) - f(0)$$

Thm. 7.2.1, p. 453.

If the functions $f, f^{'}, \ldots, f^{n}$ are (1) piecewise continuous and smooth, and (2) they are of exponential order, then -

$$\mathcal{L}\{f^{n}(t)\} =$$
$$s^{n}F(s) - s^{(n-1)}f(0) - \ldots sf^{(n-2)}(0) - f^{(n-1)}(0)$$

Corollary. Eq. 7.2.7, p. 454.