Math 2280 - Exam 2

University of Utah

Summer 2013

Name: _____

This is a one-hour exam. Please show all your work, as a worked problem is required for full points, and partial credit may be rewarded for some work in the right direction.

1. (10 points) *Converting to a First-Order System*

   Convert the following differential equation into an equivalent system of first-order equations:

$$x^{(5)} - t^2 x^{(4)} + \sin(t) x^{(3)} + x'' - 3x' + e^t x = e^{\sin t}.$$

2. (10 points) *Wronskians*

   Use the Wronskian to prove the following functions:

$$f(x) = 1 \qquad g(x) = x \qquad h(x) = x^2$$

   are linearly independent on the real line $\mathbb{R}$.

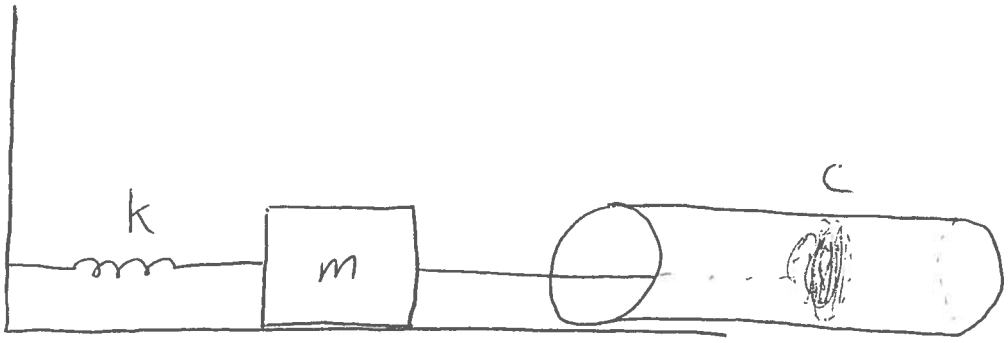
3. (10 points) *Existence and Uniqueness*

   Upon which intervals are we guaranteed there is a unique solution to the following differential equation:

$$x(x-1)y'' + e^x y' - \sin(x)y = \cos(e^{x^2+5}).$$

4. (15 points) *Mechanical Systems*

For the mass-spring-dashpot system drawn[1] below:



find the equation that describes its motion with the parameters:

$$m = 3;$$

$$c = 30;$$

$$k = 63;$$

and initial conditions:

$$x_0 = 2 \qquad\qquad v_0 = 2.$$

Is the system overdamped, underdamped, or critically damped?

[1] Not very expertly drawn.

More room for Problem 4, you'll probably need it.

5. (20 points) *Inhomogeneous Linear Differential Equations*

   Find a particular solution to the differential equation:

$$y^{(3)} + y'' = x + e^{-x}.$$

   Hint - Find the homogeneous solution first!

More room for Problem 5, if you need it.

6. (20 points) *Endpoint Values*

   The eigenvalues for the differential equation below are all nonnegative. First, determine whether $\lambda = 0$ is an eigenvalue; then find the positive eigenvalues and associated eigenfunctions.

$$y'' + \lambda y = 0;$$

$$y'(0) = 0 \qquad\qquad y(1) = 0.$$

More room for Problem 6, if you need it.

7. (15 points) *Euler's Method*

   For the differential equation:

$$\frac{dy}{dx} = y^2 - 2y + 3x^2 + 2$$

   with $y(0) = 2$ user Euler's method with step size $h = 1$ to estimate $y(2)$.

More room for Problem 7, if you need it.