Math 2280 - Exam 1

University of Utah

Summer 2013

Name: _____

This is a one-hour exam. Please show all your work, as a worked problem is required for full points, and partial credit may be rewarded for some work in the right direction.

1. (30 Points) *Differential Equation Basics*

   (a) (5 points) What is the order of the differential equation given below?[1]

$$y'' \sin(x^2) + (y'')^2 e^{x^3} + 23xy^{(3)}y^2 = 5x^6 + 7x^3 - \arctan x$$

   (b) (5 points) Is the differential equation given below linear?

$$y'' + x^2 y' + e^x y = \cos(\sin(x^2 + 3x + 2))$$

[1] Extra credit - Solve this differential equation! Just kidding. Do not attempt to solve it.

(c) (10 points) On what intervals are we guaranteed a unique solution exists for the differential equation below?

$$y' + \frac{y}{x} = \frac{x+3}{x^2-1}$$

(d) (10 points) Find the critical points for the autonomous equation:

$$\frac{dP}{dt} = kP(M-P).$$

Draw the corresponding phase diagram, and indicate if the critical points are stable, unstable, or semistable.

2. (25 points) *Separable Equations*

   Find the solution to the initial value problem given below.

$$\frac{dy}{dx} = 3x^2(y^2 + 1) \qquad\qquad y(0) = 1.$$

   Hint - The integral $\displaystyle\int \frac{du}{1+u^2} = \arctan u + C$ might be useful to you.

3. (20 points) *Exact Equations*

   Find the general solution to the differential equation given below.[2]

$$(1 + ye^{xy})dx + (2y + xe^{xy})dy = 0.$$

[2]The title of this problem is a hint.

4. (25 points) *First-Order Linear Equations*

   Find a solution to the initial value problem given below, and give the interval upon which you know the solution is unique.

$$xy' = 2y + x^3 \cos x \qquad\qquad y(\pi) = 3\pi^2.$$