Math 2280 - Assignment 8

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Section 5.4 - 1, 8, 15, 25, 33

Section 5.5 - 1, 7, 9, 18, 24

Section 5.6 - 1, 6, 10, 17, 19

Section 5.4 - Multiple Eigenvalue Solutions

5.4.1 - Find a general solution to the system of differential equations below.

$$\mathbf{x}' = \begin{pmatrix} -2 & 1 \\ -1 & -4 \end{pmatrix} \mathbf{x}$$

5.4.8 Find a general solution to the system of differential equations below.

$$\mathbf{x}' = \begin{pmatrix} 25 & 12 & 0 \\ -18 & -5 & 0 \\ 6 & 6 & 13 \end{pmatrix} \mathbf{x}$$

More room for Problem 5.4.8, if you need it.

5.4.15 - Find a general solution to the system of differential equations below.

$$\mathbf{x}' = \begin{pmatrix} -2 & -9 & 0 \\ 1 & 4 & 0 \\ 1 & 3 & 1 \end{pmatrix} \mathbf{x}$$

More room for Problem 5.4.15, if you need it.

5.4.25 - Find a general solution to the system of differential equations below. The eigenvalues of the matrix are given.

$$\mathbf{x}' = \begin{pmatrix} -2 & 17 & 4 \\ -1 & 6 & 1 \\ 0 & 1 & 2 \end{pmatrix} \mathbf{x}; \quad \lambda = 2, 2, 2.$$

More room for Problem 5.4.25, if you need it.

5.4.33 - The characteristic equation of the coefficient matrix A of the system

$$\mathbf{x}' = \begin{pmatrix} 3 & -4 & 1 & 0 \\ 4 & 3 & 0 & 1 \\ 0 & 0 & 3 & -4 \\ 0 & 0 & 4 & 3 \end{pmatrix} \mathbf{x}$$

is

$$\phi(\lambda) = (\lambda^2 - 6\lambda + 25)^2 = 0.$$

Therefore, A has the repeated complex conjugate pair $3 \pm 4i$ of eigenvalues. First show that the complex vectors

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ i \\ 0 \\ 0 \end{pmatrix} \quad \mathbf{v}_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ i \end{pmatrix}$$ [1]

form a length 2 chain $\{\mathbf{v}_1, \mathbf{v}_2\}$ associated with the eigenvalue $\lambda = 3 - 4i$. Then calculate the real and imaginary parts of the complex-valued solutions

$$\mathbf{v}_1 e^{\lambda t} \quad \text{and} \quad (\mathbf{v}_1 t + \mathbf{v}_2) e^{\lambda t}$$

to find four independent real-valued solutions of $\mathbf{x}' = A\mathbf{x}$.

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[1] Note in the textbook there's a typo in this vector.

More room for Problem 5.4.33. You'll probably need it.

Even MORE room for Problem 5.4.33, just in case.

Matrix Exponentials and Linear Systems

5.5.1 - Find a fundamental matrix for the system below, and then find a solution satisfying the given initial conditions.

$$\mathbf{x}' = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 3 \\ -2 \end{pmatrix}.$$

More room for Problem 5.5.1, if you need it.

5.5.7 - Find a fundamental matrix for the system below, and then find a solution satisfying the given initial conditions.

$$\mathbf{x}' = \begin{pmatrix} 5 & 0 & -6 \\ 2 & -1 & -2 \\ 4 & -2 & -4 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}.$$

More room for Problem 5.5.7, if you need it.

5.5.9 - Compute the matrix exponential $e^{\mathbf{A}t}$ for the system $\mathbf{x}' = \mathbf{A}\mathbf{x}$ below.

$$\begin{array}{rcrcr} x_1' & = & 5x_1 & - & 4x_2 \\ x_2' & = & 2x_1 & - & x_2 \end{array}$$

More room for Problem 5.5.9, if you need it.

5.5.18 - Compute the matrix exponential $e^{\mathbf{A}t}$ for the system $\mathbf{x}' = \mathbf{A}\mathbf{x}$ below.

$$\begin{array}{rcrcr} x_1' & = & 4x_1 & + & 2x_2 \\ x_2' & = & 2x_1 & + & 4x_2 \end{array}$$

More room for Problem 5.5.18, if you need it.

5.5.24 - Show that the matrix $\mathbf{A}$ is nilpotent and then use this fact to find the matrix exponential $e^{\mathbf{A}t}$.

$$\mathbf{A} = \begin{pmatrix} 3 & 0 & -3 \\ 5 & 0 & 7 \\ 3 & 0 & -3 \end{pmatrix}$$

More room for Problem 5.5.24, if you need it.

Nonhomogeneous Linear Systems

5.6.1 - Apply the method of undetermined coefficients to find a particular solution to the system below.

$$\begin{array}{rcrcrcr} x' & = & x & + & 2y & + & 3 \\ y' & = & 2x & + & y & - & 2 \end{array}$$

More room for Problem 5.6.1, if you need it.

5.6.6 - Apply the method of undetermined coefficients to find a particular solution to the system below.

$$\begin{array}{rcrcrcl} x' & = & 9x & + & y & + & 2e^t \\ y' & = & -8x & - & 2y & + & te^t \end{array}$$

More room for Problem 5.6.6, if you need it.

5.6.10 - Apply the method of undetermined coefficients to find a particular solution to the system below.

$$\begin{aligned} x' &= x - 2y \\ y' &= 2x - y + e^t \sin t \end{aligned}$$

More room for Problem 5.6.10, if you need it.

5.6.17 - Use the method of variation of parameters to solve the initial value problem

$$\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{f}(t),$$

$$\mathbf{x}(a) = \mathbf{x}_a.$$

The matrix exponential $e^{\mathbf{A}t}$ is given.

$$\mathbf{A} = \begin{pmatrix} 6 & -7 \\ 1 & -2 \end{pmatrix}, \quad \mathbf{f}(t) = \begin{pmatrix} 60 \\ 90 \end{pmatrix}, \quad \mathbf{x}(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

$$e^{\mathbf{A}t} = \frac{1}{6}\begin{pmatrix} -e^{-t} + 7e^{5t} & 7e^{-t} - 7e^{5t} \\ -e^{-t} + e^{5t} & 7e^{-t} - e^{5t} \end{pmatrix}$$

More room for Problem 5.6.17, if you need it.

5.6.19 - Use the method of variation of parameters to solve the initial value problem

$$\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{f}(t),$$

$$\mathbf{x}(a) = \mathbf{x}_a.$$

The matrix exponential $e^{\mathbf{A}t}$ is given.

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 2 & -2 \end{pmatrix}, \quad \mathbf{f}(t) = \begin{pmatrix} 180t \\ 90 \end{pmatrix}, \quad \mathbf{x}(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

$$e^{\mathbf{A}t} = \frac{1}{5}\begin{pmatrix} e^{-3t} + 4e^{2t} & -2e^{-3t} + 2e^{2t} \\ -2e^{-3t} + 2e^{2t} & 4e^{-3t} + e^{2t} \end{pmatrix}.$$

More room for Problem 5.6.19, if you need it.