Math 2280 - Assignment 12

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Section 9.3 - 1, 5, 8, 13, 20

Section 9.5 - 1, 3, 5, 7, 9

Please note this assignment is for *extra credit*.

Section 9.3 - Fourier Sine and Cosine Series

9.3.1 - For the given function $f(t)$ defined on the given interval find the Fourier cosine and sine series of f and sketch the graphs of the two extensions of f to which these two series converge.

$$f(t) = 1, \qquad 0 < t < \pi.$$

More room for Problem 9.3.1, if you need it.

9.3.5 - For the given function $f(t)$ defined on the given interval find the Fourier cosine and sine series of f and sketch the graphs of the two extensions of f to which these two series converge.

$$f(t) = \begin{cases} 0 & 0 < t < 1 \\ 1 & 1 < t < 2 \\ 0 & 2 < t < 3 \end{cases}$$

More room for Problem 9.3.5, if you need it.

9.3.8 - For the given function $f(t)$ defined on the given interval find the Fourier cosine and sine series of f and sketch the graphs of the two extensions of f to which these two series converge.

$$f(t) = t - t^2, \qquad 0 < t < 1$$

More room for Problem 9.3.8, if you need it.

9.3.13 - Find a formal Fourier series solution to the endpoint value problem

$$x'' + x = t \qquad\qquad x(0) = x(1) = 0.$$

More room for Problem 9.3.13, if you need it.

9.3.20 - Substitute $t = \pi/2$ and $t = \pi$ in the series

$$\frac{1}{24}t^4 = \frac{\pi^2 t^2}{12} - 2\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4}\cos nt + 2\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4}, \quad -\pi < t < \pi,$$

to obtain the summations

$$\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90},$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4} = \frac{7\pi^4}{720},$$

and

$$1 + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + \cdots = \frac{\pi^4}{96}.$$

More room for Problem 9.3.20, if you need it.

Section 9.5 - Heat Conduction and Separation of Variables

9.5.1 - Solve the boundary value problem $u_t = 3u_{xx}$, $0 < x < \pi$, $t > 0$; $u(0,t) = u(\pi,t) = 0$, $u(x,0) = 4\sin 2x$.

More room for Problem 9.5.1, if you need it.

9.5.3 - Solve the boundary value problem $u_t = 2u_{xx}$, $0 < x < 1$, $t > 0$; $u(0,t) = u(1,t) = 0$, $u(x,0) = 5\sin \pi x - \frac{1}{5}\sin 3\pi x$.

More room for Problem 9.5.3, if you need it.

9.5.5 - Solve the boundary value problem $u_t = 2u_{xx}$, $0 < x < 3$, $t > 0$; $u_x(0,t) = u_x(3,t) = 0$, $u(x,0) = 4\cos\frac{2}{3}\pi x - 2\cos\frac{4}{3}\pi x$.

More room for Problem 9.5.5, if you need it.

9.5.7 - Solve the boundary value problem $3u_t = u_{xx}$, $0 < x < 2$, $t > 0$; $u_x(0,t) = u_x(2,t) = 0$, $u(x,0) = \cos^2 2\pi x$.

More room for Problem 9.5.7, if you need it.

9.5.9 - Solve the boundary value problem $10u_t = u_{xx}$, $0 < x < 5$, $t > 0$; $u(0,t) = u(5,t) = 0$, $u(x,0) = 25$.

More room for Problem 9.5.9, if you need it.