Math 2280 - Assignment 11

Dylan Zwick

Summer 2013

Section 7.6 - 1, 6, 11, 14, 15

Section 9.1 - 1, 8, 11, 13, 21

Section 9.2 - 1, 9, 15, 17, 20

Inpulses and Delta Functions

7.6.1 - Solve the initial value problem

$$x'' + 4x = \delta(t);$$

$$x(0) = x'(0) = 0,$$

and graph the solution $x(t)$.

7.6.6 - Solve the initial value problem

$$x'' + 9x = \delta(t - 3\pi) + \cos 3t;$$

$$x(0) = x'(0) = 0,$$

and graph the solution $x(t)$.

7.6.11 - Apply Duhamel's principle to write an integral formula for the solution of the initial value problem

$$x'' + 6x' + 8x = f(t);$$

$$x(0) = x'(0) = 0.$$

7.6.14 - Verify that $u'(t-a) = \delta(t-a)$ by solving the problem

$$x' = \delta(t-a);$$
$$x(0) = 0$$

to obtain $x(t) = u(t-a)$.

7.6.15 - This problem deals with a mass m on a spring (with constant k) that receives an impulse $p_0 = mv_0$ at time $t = 0$. Show that the initial value problems

$$mx'' + kx = 0;$$

$$x(0) = 0, x'(0) = v_0$$

and

$$mx'' + kx = p_0\delta(t);$$

$$x(0) = 0, x'(0) = 0$$

have the same solution. Thus the effect of $p_0\delta(0)$ is, indeed, to impart to the particle an initial momentum p_0.

More space, if you need it, for Problem 7.6.15.

Section 9.1 - Periodic Functions and Trigonometric Series

9.1.1 - Sketch the graph of the function f defined for all t by the given formula, and determine whether it is periodic. If so, find its smallest period.

$$f(t) = \sin 3t.$$

9.1.8 - Sketch the graph of the function f defined for all t by the given formula, and determine whether it is periodic. If so, find its smallest period.

$$f(t) = \sinh \pi t.$$

9.1.11 - The value of a period 2π function $f(t)$ in one full period is given below. Sketch several periods of its graph and find its Fourier series.

$$f(t) = 1, \qquad\qquad -\pi \leq t \leq \pi.$$

9.1.13 - The value of a period 2π function $f(t)$ in one full period is given below. Sketch several periods of its graph and find its Fourier series.

$$f(t) = \begin{cases} 0 & -\pi < t \leq 0 \\ 1 & 0 < t \leq \pi \end{cases}$$

9.1.21 - The value of a period 2π function $f(t)$ in one full period is given below. Sketch several periods of its graph and find its Fourier series.

$$f(t) = t^2, \qquad\qquad -\pi \leq t < \pi$$

Section 9.2 - General Fourier Series and Convergence

9.2.1 - The values of a periodic function $f(t)$ in one full period are given below; at each discontinuity the value of $f(t)$ is that given by the average value condition. Sketch the graph of f and find its Fourier series.

$$f(t) = \begin{cases} -2 & -3 < t < 0 \\ 2 & 0 < t < 3 \end{cases}$$

9.2.9 - The values of a periodic function $f(t)$ in one full period are given below; at each discontinuity the value of $f(t)$ is that given by the average value condition. Sketch the graph of f and find its Fourier series.

$$f(t) = t^2, \qquad -1 < t < 1$$

9.2.15 -

(a) - Suppose that f is a function of period 2π with $f(t) = t^2$ for $0 < t < 2\pi$. Show that

$$f(t) = \frac{4\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{\cos nt}{n^2} - 4\pi \sum_{n=1}^{\infty} \frac{\sin nt}{n}$$

and sketch the graph of f, indicating the value at each discontinuity.

(b) - Deduce the series summations

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

and

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$$

from the Fourier series in part (a).

More room for Problem 9.2.15, if you need it.

9.2.17 -

(a) - Supose that f is a funciton of period 2 with $f(t) = t$ for $0 < t < 2$. Show that

$$f(t) = 1 - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\sin n\pi t}{n}$$

and sketch the graph of f, indicating the value at each discontinuity.

(b) - Substitute an appropriate value of t to deduce *Leibniz's series*

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}.$$

More room for Problem 9.2.17, if you need it.

9.2.20 - Derive the Fourier series given below, and graph the period 2π function to which the series converges.

$$\sum_{n=1}^{\infty} \frac{\cos nt}{n^2} = \frac{3t^2 - 6\pi t + 2\pi^2}{12} \qquad (0 < t < 2\pi)$$

More room for Problem 9.2.20, if you need it.