Math 2280 - Assignment 10

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Summer 2013

Section 7.3 - 3, 8, 19, 24, 30, 33

Section 7.4 - 1, 5, 10, 19, 31

Section 7.5 - 1, 6, 15, 21, 26

Section 7.3 - Translation and Partial Fractions

7.3.3 - Apply the translation theorem to find the Laplace transform of the function

$$f(t) = e^{-2t} \sin 3\pi t.$$

7.3.8 - Apply the translation theorem to find the inverse Laplace transform of the function

$$F(s) = \frac{s+2}{s^2+4s+5}.$$

7.3.19 - Use partial fractions to find the inverse Laplace transform of the function

$$F(s) = \frac{s^2 - 2s}{s^4 + 5s^2 + 4}.$$

7.3.24 - Use the factorization

$$s^4 + 4a^4 = (s^2 - 2as + 2a^2)(s^2 + 2as + 2a^2)$$

to derive the inverse Laplace transform

$$\mathcal{L}^{-1}\left\{\frac{s}{s^4 + 4a^4}\right\} = \frac{1}{2a^2}\sinh at \sin at.$$

More room for Problem 7.3.24 in case you need it.

7.3.30 - Use Laplace transforms to solve the initial value problem

$$x'' + 4x' + 8x = e^{-t} \quad x(0) = x'(0) = 0.$$

7.3.33 - Use Laplace transforms to solve the initial value problem

$$x^{(4)} + x = 0 \quad x(0) = x'(0) = x''(0) = 0, x^{(3)}(0) = 1.$$

More room for Problem 7.3.33 in case you need it.

Section 7.4 - Derivatives, Integrals, and Products of Transforms

7.4.1 - Find the convolution $f(t) * g(t)$ of the functions

$$f(t) = t, \quad g(t) = 1.$$

7.4.5 - Find the convolution $f(t) * g(t)$ of the functions

$$f(t) = g(t) = e^{at}.$$

7.4.10 - Apply the convolution theorem to find the inverse Laplace transform of the function

$$F(s) = \frac{1}{s^2(s^2 + k^2)}.$$

7.4.19 - Find the Laplace transform of the function

$$f(t) = \frac{\sin t}{t}.$$

7.4.31 - Transform the given differential equation to find a nontrivial solution such that $x(0) = 0$.

$$tx'' - (4t + 1)x' + 2(2t + 1)x = 0.$$

More room for Problem 7.4.31, if you need it.

Section 7.5 - Periodic and Piecewise Continuous Input Functions

7.5.1 - Find the inverse Laplace transform $f(t)$ of the function

$$F(s) = \frac{e^{-3s}}{s^2}.$$

7.5.6 - Find the inverse Laplace transform $f(t)$ of the function

$$F(s) = \frac{se^{-s}}{s^2 + \pi^2}.$$

7.5.15 - Find the Laplace transform of the function

$$f(t) = \sin t \text{ if } 0 \leq t \leq 3\pi; \, f(t) = 0 \text{ if } t > 3\pi.$$

7.5.21 - Find the Laplace transform of the function

$$f(t) = t \text{ if } t \leq 1;\ f(t) = 2 - t \text{ if } 1 \leq t \leq 2;\ f(t) = 0 \text{ if } t > 2.$$

7.5.26 - Apply Theorem 2 to show that the Laplace transform of the saw-tooth function $f(t)$ pictured below is

$$F(s) = \frac{1}{as^2} - \frac{e^{-as}}{s(1 - e^{-as})}.$$

More room for Problem 7.5.26, if you need it.