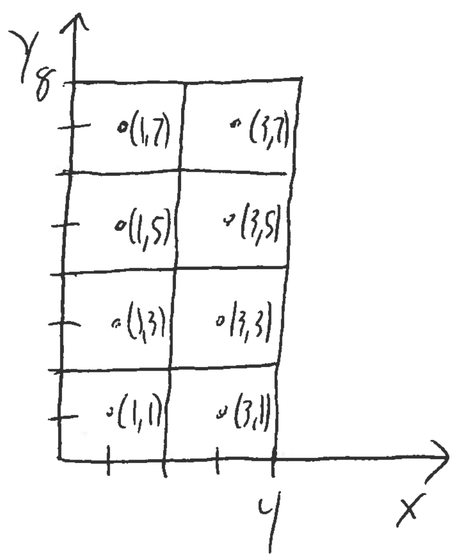
Math2210 Quiz 9 (Section 13.1, 13.2, 13.3) Summer, 2012 Dylan Zwick

Name Key Date 7-25-2012

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. *(15 points)* Take the region $R=\{(x,y):0\le x\le 4, 0\le y\le 8\}$, the function $f(x,y)=x^2+2xy-y+x^3$, and the partition P of R into eight equal squares by the lines x = 2, y = 2, y = 4, and y = 6.

Approximate $\iint_R f(x,y)\,dA$ by calculating the corresponding Riemann sum $\sum_{k=1}^{8} f(\bar{x}_k,\bar{y}_k)\Delta A_k$, assuming that $(\bar{x}_k,\bar{y}_k)$ are the centers of the eight squares.



$f(1,1)=3$ $f(3,1)=41$

$f(1,3)=5$ $f(3,3)=51$

$f(1,5)=7$ $f(3,5)=61$

$f(1,7)=9$ $f(3,7)=71$

$$\sum_{k=1}^{8} f(\bar{x}_k,\bar{y}_k)\Delta A_k$$

$$=4(3+5+7+9+41+51+61+71)$$

$$=4(248)=992$$

Answer: $\boxed{992}$

2. Evaluate the following integrals.

(a) $\int_0^3\int_1^3 (x^2+x+y^2+1)\,dx\,dy$. *(10 points)*

$$= \int_0^3 \left[\frac{x^3}{3} + \frac{x^2}{2} + xy^2 + x \Big|_1^3\right] dy = \int_0^3 \Big[\left(9 + \frac{9}{2} + 3y^2 + 3\right) - \left(\frac{1}{3} + \frac{1}{2} + y^2 + 1\right)\Big] dy$$

$$= \int_0^3 \left[\left(9 + \frac{9}{2} + 3 - \frac{1}{3} - \frac{1}{2} - 1\right) + 2y^2\right] dy = \int_0^3 \left[\left(\frac{66+27-5}{6}\right) + 2y^2\right] dy$$

$$= \int_0^3 \left[\frac{88}{6} + 2y^2\right] dy = \int_0^3 \left(\frac{44}{3} + 2y^2\right) dy = \frac{44}{3}y + \frac{2}{3}y^3 \Big|_0^3$$

$$= \frac{44}{3}(3) + 2(9) = 44 + 18 = 62$$

Answer : $\boxed{62}$

(b) $\int_{-1}^{1}\int_{-x}^{2-x^2} (x+y)\,dy\,dx$. *(15 points)*

$$= \int_{-1}^{1} \left(xy + \frac{y^2}{2} \Big|_{-x}^{2-x^2}\right) dx = \int_{-1}^{1} \Big[\left(x(2-x^2) + \frac{(2-x^2)^2}{2}\right) - \left(-x^2 + \frac{x^2}{2}\right)\Big] dy$$

$$= \int_{-1}^{1} \left[2x - x^3 + \frac{4 - 4x^2 + x^4}{2} + \frac{x^2}{2}\right] dx = \int_{-1}^{1} \left[2 + 2x - \frac{3x^2}{2} - x^3 + \frac{x^4}{2}\right] dx$$

$$= \int_{-1}^{1} \left[2 - \frac{3}{2}x^2 + \frac{x^4}{2}\right] dx = 2\int_0^1 \left[2 - \frac{3}{2}x^2 + \frac{x^4}{2}\right] dx$$

Even part

$$= 2\left[2x - \frac{x^3}{2} + \frac{x^5}{10} \Big|_0^1\right] = 2\left(2 - \frac{1}{2} + \frac{1}{10}\right)$$

$$= 2\left(\frac{16}{10}\right) = \frac{16}{5}$$

Answer: $\boxed{16/5}$