Math2210 Quiz 8 (Sections 12.8-12.9) Summer, 2012 Dylan Zwick

Name Key Date 7-23-2012

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. *(20 Points)* Express the number 42 as the sum of three positive numbers such that the product of these three numbers is a maximum.

$x+y+z=42 \qquad f(x,y,z)=xyz$

$f(x,y)=xy(42-x-y)=42xy-x^2y-xy^2$

$f_x=42y-2xy-y^2 \qquad f_y=42x-x^2-2xy$

$42y-2xy-y^2=0 \Rightarrow x=\frac{y^2-42y}{-2y}=21-\frac{y}{2}$

$42x-x^2-2xy=0$

$42\left(21-\frac{y}{2}\right)-\left(21-\frac{y}{2}\right)^2-2\left(21-\frac{y}{2}\right)y=882-21y-441+21y-\frac{y^2}{4}-42y+y^2$

$\Rightarrow \frac{3}{4}y^2-42y+441=0$

$y=\frac{42\pm\sqrt{(42)^2-4(3/4)(441)}}{2(3/4)}$

$=\frac{42\pm\sqrt{1762-1762(3/4)}}{2(3/4)} = \cancel{21}$

$=\frac{42\pm\sqrt{441}}{3/2}=\frac{(42-21)2}{3}=\boxed{14}$

$x=21-\frac{14}{2}=\boxed{14}$

$z=42-14-14=\boxed{14}$

$f_{xx}=-2y$

$f_{yy}=-2x$

$f_{xy}=42-2x-2y$

$D(14,14)=4(14)(14)-(-14)^2$

$=3(14)^2=588>0$

$f_{xy}(14,14)=-2(14)=-28<0$

So, it's a max.

Answer: $14+14+14=42$

2. *(20 Points)* Find the maximum volume of a closed rectangular box with faces parallel to the coordinate planes inscribed in the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1 \ .$$

Let x, y, z be the coordinates of the vertex of the box in the first octant. We want to maximize

$f(x,y,z)=8xyz$ subject to $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

$\nabla f = \langle 8yz, 8xz, 8xy \rangle \quad \nabla g = \left\langle \frac{2x}{a^2}, \frac{2y}{b^2}, \frac{2z}{c^2} \right\rangle$

$8yz = \frac{2\lambda x}{a^2} \quad 8xz = \frac{2\lambda y}{b^2} \quad 8xy = \frac{2\lambda z}{c^2} \quad \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

$\lambda = \frac{4a^2yz}{x} \Rightarrow 8xz = \frac{8a^2y^2z}{b^2x}, \quad 8xy = \frac{8a^2yz^2}{c^2x}, \quad \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

$z \neq 0$, so

$b^2x^2 = a^2y^2$

$\Rightarrow c^2x^2 = a^2z^2$

$\Rightarrow x^2 = \frac{a^2y^2}{b^2} \Rightarrow \frac{a^2c^2y^2}{b^2} = a^2z^2 \Rightarrow y^2 = \frac{b^2z^2}{c^2} \Rightarrow \frac{z^2}{c^2}+\frac{z^2}{c^2}+\frac{z^2}{c^2}=1$

$x^2 = \frac{a^2z^2}{c^2}$

$\Rightarrow z^2 = \frac{c^2}{3} \Rightarrow z = \frac{c}{\sqrt{3}}, \quad x = \frac{a}{\sqrt{3}}, \quad y = \frac{b}{\sqrt{3}}$

$V = 8\left(\frac{abc}{3\sqrt{3}}\right)$

Answer: $\boxed{\frac{8abc}{3\sqrt{3}}}$ with $x = \frac{a}{\sqrt{3}}$ $y = \frac{b}{\sqrt{3}}$ $z = \frac{c}{\sqrt{3}}$