Math2210 Quiz 7 (Sections 12.6-12.7) Summer, 2012 Dylan Zwick

Name Key Date 7-19-2012

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. *(15 points)* Find $\frac{\partial w}{\partial t}$ using the chain rule for $w=e^{xy+z}$ given $x=s+t$, $y=s-t$, and $z=t^2$. Express your answer in terms of s and t.

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial t}$$

$$\frac{\partial w}{\partial x} = y e^{xy+z} \quad \frac{\partial w}{\partial y} = x e^{xy+z} \quad \frac{\partial w}{\partial z} = e^{xy+z}$$

$$\frac{\partial x}{\partial t} = 1 \quad \frac{\partial y}{\partial t} = -1 \quad \frac{\partial z}{\partial t} = 2t$$

$$\Rightarrow \frac{\partial w}{\partial t} = y e^{xy+z} - x e^{xy+z} + 2t e^{xy+z}$$

$$= (y-x+2t) e^{xy+z}$$

$$y-x = (s-t)-(s+t) = -2t$$

$$\Rightarrow = (-2t+2t) e^{xy+z} = 0 e^{xy+z} = 0$$

Answer: $\boxed{0}$

2. *(10 points)* Find $\frac{dw}{dt}$ using the chain rule for $\ln\frac{x}{y}$ given $x=\tan t$, and $y=sec^2 t$. Express your answer in terms of t.

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt}$$

$$\frac{\partial w}{\partial x} = \frac{\frac{1}{y}}{\frac{x}{y}} = \frac{1}{x}$$

$$\frac{\partial w}{\partial y} = \frac{-\frac{x}{y^2}}{\frac{x}{y}} = -\frac{1}{y}$$

$$\frac{dx}{dt} = \sec^2 t \quad \frac{dy}{dt} = 2\sec^2 t \tan t$$

$$= \frac{1}{x}\sec^2 t - \frac{1}{y} 2\sec t \tan t$$

$$= \frac{\sec^2 t}{\tan t} - \frac{2\sec^2 t\tan t}{\sec^2 t}$$

$$= \cancel{\frac{\cos t}{\sin t} - 2\sin t} \; 2\tan t$$

$$= \frac{\sec t}{\sin t} - 2\tan t$$

Answer : $\boxed{\frac{\sec t}{\sin t} - 2\tan t}$

3. *(15 points)* Find a point on the surface $z=2x^2+3y^2$ where the tangent plane is parallel to the plane $8x-3y-z=0$.

$$2x^2+3y^2-z=0 = f(x,y,z)$$

$$\nabla f = \langle 4x, 6y, -1\rangle = k\langle 8,-3,-1\rangle$$

$$k=1 \quad x=2 \quad y=-\frac{1}{2}$$

$$z = 2(2^2)+3\left(-\frac{1}{2}\right)^2 = 8+\frac{3}{4} = \frac{32}{4}+\frac{3}{4} = \frac{35}{4}$$

Answer: $\left(2, -\frac{1}{2}, \frac{35}{4}\right)$