Math2210 Quiz 6 (Sections 12.3-12.5) Summer, 2012 Dylan Zwick

Name Key Date 7-17-2012

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. *(15 points)* Find the limit of $\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$ or state and explain why it does not exist.

$x = r\cos\theta$

$y = r\sin\theta$

$$\lim_{r\to 0} \frac{r^2\cos\theta\sin\theta}{r}$$

$$= \lim_{r\to 0} r\cos\theta\sin\theta$$

$$\lim_{r\to 0} -r \le \lim_{r\to 0} r\cos\theta\sin\theta \le \lim_{r\to 0} r$$

$$\Rightarrow 0 \le \lim_{r\to 0} r\cos\theta\sin\theta \le 0$$

$$\Rightarrow \lim_{r\to 0} r\cos\theta\sin\theta = 0$$

Answer : 0

2. *(10 points)* Find the equation of the tangent plane of $f(x,y)=x^3y+3xy^2$ at the point at $\mathbf{p}=(2,-2)$?

$$f(2,-2) = 2^3(-2) + 3(2)(-2)^2 = 8$$

$$\nabla f = \langle 3x^2y + 3y^2, x^3 + 6xy \rangle$$

$$\nabla f(2,-2) = \langle 3(2)^2(-2) + 3(-2)^2, 2^3 + 6(2)(-2) \rangle$$

$$= \langle -12, -16 \rangle$$

$$T(\vec{p}) = z = 8 + \langle -12, -16 \rangle \cdot \langle x-2, y-(-2) \rangle$$

$$= 8 - 12x + 24 - 16y - 32$$

$$z = -12x - 16y \Rightarrow 12x + 16y + z = 0$$

Answer: $\boxed{12x + 16y + z = 0}$

3. *(15 points)* Find ∇f given $f(x,y)=x^2-3xy+2y^2$. Use this to find the directional derivative of f at the point $(-1,2)$ in the direction of the vector $2\mathbf{i}-\mathbf{j}$.

$$\nabla f = \langle 2x-3y,\ 4y-3x \rangle$$

$$\nabla f(-1,2) = \cancel{\langle -8, 10 \rangle}\ \langle -8, 11 \rangle$$

$$\vec{a} = 2\hat{i} - \hat{j}$$

$$\hat{a} = \frac{\vec{a}}{\|\vec{a}\|} = \frac{\langle 2,-1 \rangle}{\sqrt{5}} = \left\langle \frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}} \right\rangle$$

$$D_{\hat{a}} f(-1,2) = \langle -8, 11 \rangle \cdot \left\langle \frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}} \right\rangle$$

$$= -\frac{16}{\sqrt{5}} - \frac{11}{\sqrt{5}} = -\frac{27}{\sqrt{5}}$$

∇f : $\langle 2x-3y,\ 4y-3x \rangle$

Directional Derivative : $-\frac{27}{\sqrt{5}}$