

Name Key Date 7-16-2012

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. Name the type of quadric surface given by  $9x^2 + 4y^2 + 25z^2 - 16 = 0$ .

$$9x^2 + 4y^2 + 25z^2 = 16$$

$$\frac{9x^2}{16} + \frac{4y^2}{16} + \frac{25z^2}{16} = 1$$

All second order, all positive, so ellipsoid.

Type of surface: Ellipsoid

2. Name the type of quadric surface given by  $16x^2 - 4y^2 - 36z^2 + 90 = 9$ .

$$16x^2 - 4y^2 - 36z^2 + 90 = 9$$

$$16x^2 - 4y^2 - 36z^2 = -81$$

$$\frac{4}{81}y^2 + \frac{36}{81}z^2 - \frac{16}{18}x^2 = 1$$

All second order, ~~at~~ two positive. So, hyperboloid of one sheet, opening along the x-axis.

Type of surface: Hyperboloid of One Sheet

3. a) Change  $(4, \frac{\pi}{3}, \frac{3\pi}{4})$  from cylindrical coordinates to Cartesian.

$$x = r \cos \theta = 4 \cos\left(\frac{\pi}{3}\right) = 4\left(\frac{1}{2}\right) = 2$$

$$y = r \sin \theta = 4 \sin\left(\frac{\pi}{3}\right) = 4\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$$

$$z = \frac{3\pi}{4}$$

Answer:  $(2, 2\sqrt{3}, \frac{3\pi}{4})$

- b) Change  $(4\sqrt{3}, -4, 6)$  from Cartesian coordinates to cylindrical.

$$z = 6$$

$$r = \sqrt{(4\sqrt{3})^2 + (-4)^2} = \sqrt{48 + 16} = \sqrt{64} = 8$$

$$\theta = \tan^{-1}\left(\frac{-4}{4\sqrt{3}}\right) = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -\pi/6$$

Answer:  $(8, -\pi/6, 6)$

4. Change  $x^2 + y^2 - 9z^2 = 81$  to the following coordinates:

- a) Spherical

$$\rho^2 \cos^2 \theta \sin^2 \phi + \rho^2 \sin^2 \theta \sin^2 \phi - 9\rho^2 \cos^2 \phi = 81$$

$$= \rho^2 (\sin^2 \phi - 9\cos^2 \phi) = 81$$

Answer:  $\rho^2 = \frac{81}{\sin^2 \phi - 9\cos^2 \phi}$

- b) Cylindrical

$$x^2 + y^2 = r^2$$

$$r^2 - 9z^2 = 81$$

$$r^2 = 9(z^2 + 9)$$

Answer:  $r^2 = 9(z^2 + 9)$