

Name _____ Date _____

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. If $\mathbf{a} = \langle 3, 3, 2 \rangle$, $\mathbf{b} = \langle -1, 1, 2 \rangle$ and $\mathbf{c} = \langle -1, 2, 4 \rangle$,

(a) find $\mathbf{a} \cdot (\mathbf{b} - \mathbf{c})$.

$$\begin{aligned}\vec{b} - \vec{c} &= \langle -1, 1, 2 \rangle - \langle -1, 2, 4 \rangle \\ &= \langle 0, -1, -2 \rangle\end{aligned}$$

$$\vec{a} \cdot (\vec{b} - \vec{c}) = \langle 3, 3, 2 \rangle \cdot \langle 0, -1, -2 \rangle = 0 - 3 - 4 = -7$$

(b) find $|\mathbf{a}|(\mathbf{b} \times \mathbf{c})$

$$\mathbf{a} \cdot (\mathbf{b} - \mathbf{c}) = \underline{\boxed{-7}}$$

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 2 \\ -1 & 2 & 4 \end{vmatrix} = 0\hat{i} + 2\hat{j} + (-1)\hat{k}$$

$$|\vec{a}| = \sqrt{3^2 + 3^2 + 2^2} = \sqrt{22}$$

$$|\vec{a}|(\vec{b} \times \vec{c}) = \langle 0, 2\sqrt{22}, -\sqrt{22} \rangle$$

$$|\mathbf{a}|(\mathbf{b} \times \mathbf{c}) = \underline{2\sqrt{22}\hat{j} - \sqrt{22}\hat{k}}$$

2. Find parametric equations for the line of intersection of the planes
 $5x-3y-2z = 5$ and $x + y + 2z = 3$.

$$\vec{n}_1 = \langle 5, -3, -2 \rangle$$

$$\vec{n}_2 = \langle 1, 1, 2 \rangle$$

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -3 & -2 \\ 1 & 1 & 2 \end{vmatrix} = -4\hat{i} - 12\hat{j} + 8\hat{k}$$

$$\begin{aligned} x = 0 \quad \begin{cases} -3y - 2z = 5 \\ y + 2z = 3 \end{cases} &\Rightarrow \begin{aligned} -2y &= 8 \Rightarrow y = -4 \\ -4 + 2z &= 3 \\ 2z &= 7 \Rightarrow z = \frac{7}{2} \end{aligned} \end{aligned}$$

Point $(0, -4, \frac{7}{2})$

Direction $\langle -4, -12, 8 \rangle$

Parametric Equations: $x(t) = -4t$ $y(t) = -4 - 12t$ $z(t) = \frac{7}{2} + 8t$

3. For the particle with position vector $\vec{r}(t) = (3t+4)\hat{i} + e^t\hat{j} + \sin(2t)\hat{k}$ calculate the velocity $\vec{v}(t)$ and the acceleration $\vec{a}(t)$.

$$\vec{v}(t) = \vec{r}'(t) = 3\hat{i} + e^t\hat{j} + 2\cos(2t)\hat{k}$$

$$\vec{a}(t) = \vec{v}'(t) = \vec{r}''(t) = 0\hat{i} + e^t\hat{j} - 4\sin(2t)\hat{k}$$

$$\vec{v}(t) = \underline{3\hat{i} + e^t\hat{j} + 2\cos(2t)\hat{k}}$$

$$\vec{a}(t) = \underline{e^t\hat{j} - 4\sin(2t)\hat{k}}$$