

Name key Date 7-31-2012

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. (15 points) Determine whether  $\mathbf{F}(x, y) = (e^x \sin y + 1)\mathbf{i} + (e^x \cos y + 6)\mathbf{j}$  is conservative. If so, find a function  $f$  such that  $\mathbf{F} = \nabla f$ . If not, state that  $\mathbf{F}$  is not conservative.

$$\frac{\partial M}{\partial y} = e^x \cos y \quad \frac{\partial N}{\partial x} = e^x \cos y$$

So,  $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ , so  $\boxed{\vec{F} \text{ is conservative.}}$

$$\frac{\partial f}{\partial x} = e^x \sin y + 1 \Rightarrow f(x, y) = e^x \sin y + x + C(y)$$

$$\frac{\partial f}{\partial y} = e^x \cos y + C'(y) = e^x \cos y + 6$$

$$\Rightarrow C'(y) = 6 \Rightarrow C(y) = 6y + C.$$

So,  $\boxed{f = e^x \sin y + x + 6y + C}$

Conservative:  True or  False (circle one)

4. If conservative,  $f = \underline{e^x \sin y + x + 6y + C}$   
(Any choice of  $C$  is also acceptable.)

2. (15 points) For the vector field given above,

$$\mathbf{F}(x, y) = (e^x \sin y + 1)\mathbf{i} + (e^x \cos y + 6)\mathbf{j},$$

calculate the line integral  $\int_C \mathbf{F} \cdot d\mathbf{r}$  from  $(0, 0)$  to  $(2, \frac{\pi}{6})$

using any method you wish.

$$\begin{aligned} \int_{(0,0)}^{(2, \pi/6)} \vec{F} \cdot d\vec{r} &= f(2, \frac{\pi}{6}) - f(0, 0) \\ &= e^2 \sin(\frac{\pi}{6}) + 2 + 6(\frac{\pi}{6}) - 0 \\ &= \frac{e^2}{2} + 2 + \pi \end{aligned}$$

Answer  $\frac{e^2}{2} + 2 + \pi$

3. (10 points) What... is your favorite color?

Answer: Yellow

(Answers will, of course, vary.)