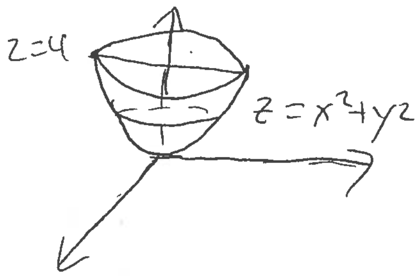
Math2210 Quiz 11 (Sections 13.8, 14.1) Summer, 2012 Dylan Zwick

Name: Key Date: 7-30-2012

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. *(18 points)* Calculate the volume of the solid bounded by the paraboloid $z=x^2+y^2$ and the plane $z=4$.



$$\int_0^{2\pi}\int_0^{2}\int_{r^2}^{4} r\,dz\,dr\,d\theta$$

$$=\int_0^{2\pi}\int_0^{2} zr\Big|_{r^2}^{4}\,dr\,d\theta$$

$$=\int_0^{2\pi}\int_0^{2}(4r-r^3)\,dr\,d\theta$$

$$=\int_0^{2\pi} 2r^2-\frac{r^4}{4}\Big|_0^{2}\,d\theta$$

$$=\int_0^{2\pi}(8-4)\,d\theta=\int_0^{2\pi}4\,d\theta$$

$$=4\theta\Big|_0^{2\pi}=8\pi$$

Answer: $\boxed{8\pi}$

2. *(14 points)* Calculate div **F** and curl **F** for the vector field

$$\boldsymbol{F}(x,y,z)=e^{x}\cos(y)\,\boldsymbol{i}+e^{x}\sin(y)\,\boldsymbol{j}+z\,\boldsymbol{k}$$

$$\nabla\cdot\vec{F} = e^{x}\cos y + e^{x}\cos y + 1 = 1+2e^{x}\cos y$$

$$\nabla\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^{x}\cos y & e^{x}\sin y & z \end{vmatrix}$$

$$= (0-0)\hat{i} + (0-0)\hat{j} + (e^{x}\sin y + e^{x}\sin y)\hat{k}$$

div **F**: $1+2e^{x}\cos y$

curl **F**: $2e^{x}\sin y\,\hat{k}$

3. *(2 points each)* If f(x,y,z) is a scalar function and **F**(x,y,z) is a vector field, which of the following make sense (circle one):

a) $\nabla\cdot\nabla(f)$ — Makes sense. — Does not make sense.

b) div(grad(f)) — Makes sense. — Does not make sense.

c) grad(div(**F**)) — Makes sense. — Does not make sense.

d) $\nabla\times\nabla(f)$ — Makes sense. — Does not make sense.