Math2210 Quiz 10 (Sections 13,4, 13.6, 13.7) Summer, 2012 Dylan Zwick

Name Key Date 7-26-2012

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. *(20 points)* Calculate the surface area of the part of the surface $z=\sqrt{4-y^2}$ in the first octant that is directly above the circle $x^2+y^2=4$ in the xy-plane.

$$z=\sqrt{4-y^2} \qquad \frac{\partial z}{\partial x}=0 \qquad \frac{\partial z}{\partial y}=-\frac{y}{\sqrt{4-y^2}}$$

$$SA=\iint_R \sqrt{1+0^2+\left(-\frac{y}{\sqrt{4-y^2}}\right)^2}\,dA=\iint_R \sqrt{1+\frac{y^2}{4-y^2}}\,dA$$

$$=\iint_R \frac{2}{\sqrt{4-y^2}}\,dA$$

2 Ways:

Cartesian

$$=\int_0^2\int_0^{\sqrt{4-y^2}}\frac{2}{\sqrt{4-y^2}}\,dx\,dy$$

$$=\int_0^2 \frac{2\sqrt{4-y^2}}{\sqrt{4-y^2}}\,dy$$

$$=\int_0^2 2\,dy=2y\Big|_0^2$$

$$=\boxed{4}$$

Polar:

$$\int_0^{\pi/2}\int_0^2 \frac{2r}{\sqrt{4-r^2\sin^2\theta}}\,dr\,d\theta$$

$$u=4-r^2\sin^2\theta$$
$$du=-2r\sin^2\theta\,dr$$

$$-\int_0^{\pi/2}\int_4^{4(1-\sin^2\theta)}\frac{du}{\sin^2\theta\sqrt{u}}\,d\theta$$

$$=\int_0^{\pi/2}\int_{4\cos^2\theta}^{4}\frac{du}{\sin^2\theta\sqrt{u}}\,d\theta=2\int_0^{\pi/2}\frac{2-2\cos\theta}{\sin^2\theta}\,d\theta$$

$$=4\int_0^{\pi/2}\frac{1-\cos\theta}{1-\cos^2\theta}\,d\theta=4\int_0^{\pi/2}\frac{1}{1+\cos\theta}\,d\theta$$

$$=4\tan\left(\frac{\theta}{2}\right)\Big|_0^{\pi/2}=4\tan\left(\frac{\pi}{4}\right)-4\tan(0)$$

$$=\boxed{4}$$

Answer: 4

2. *(10 points) Evaluate the iterated integral* $\int_0^2 \int_1^z \int_0^{\sqrt{x/z}} 2xyz\,dydxdz$.

$$\int_0^2 \int_1^z xy^2z \Big|_0^{\sqrt{x/z}} dxdz = \int_0^2 \int_1^z x^2\,dxdz$$

$$= \int_0^2 \frac{x^3}{3}\Big|_1^z dz = \int_0^2 \left(\frac{z^3}{3} - \frac{1}{3}\right)dz$$

$$= \frac{z^4}{12} - \frac{z}{3}\Big|_0^2 = \frac{16}{12} - \frac{2}{3} = \frac{4}{3} - \frac{2}{3}$$

Answer: $\frac{2}{3}$

3. *(10 points)* Calculate $\iint_S e^{x^2+y^2}\,dA$, where *S* is the region enclosed by $x^2+y^2=4$.

$$= \int_0^{2\pi}\int_0^2 e^{r^2} r\,dr\,d\theta \qquad \begin{aligned} u &= r^2 \\ du &= 2r\,dr \end{aligned}$$

$$= \frac{1}{2}\int_0^{2\pi}\int_0^4 e^u\,du\,d\theta = \frac{1}{2}\int_0^{2\pi} e^u\Big|_0^4 d\theta$$

$$= \frac{1}{2}\int_0^{2\pi}(e^4-1)\,d\theta = \frac{2\pi}{2}(e^4-1)$$

$$= \pi(e^4-1)$$

Answer : $\boxed{\pi(e^4-1)}$