Math2210 Midterm 3 Summer, 2012 Dylan Zwick

Name ______________________________ Date _____________

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. Evaluate the integrals.

(a) (10 points) $\int_1^2 \int_0^3 xy + y^2 \, dy \, dx$.

Answer 1(a): ____

(b) (15 points) $\int_0^{\pi/2} \int_{\sin 2z}^{0} \int_0^{2yz} \sin\left(\frac{x}{y}\right) dx \, dy \, dz$.

Answer 1(b): ____

(Note: This is #1 continued!)

(c) (20 points) $\int_0^2 \int_y^2 e^{-x^2}\, dx\, dy$

Answer 1(c): ____

2. (15 points) Calculate the area of the region inside the circle $r = 4\cos\theta$ and outside the circle $r = 2$.

   Hint: $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$.

Answer 2: ____

3. (15 points) If $R=((x,y):0\leqslant x\leqslant 6, 0\leqslant y\leqslant 6)$ and P is the partition of R into nine equal squares by the lines x = 2, x = 4, y = 2, and y = 4. Approximate $\iint_R f(x,y)\,dA$ by calculating the corresponding Riemann sum $\sum_{k=1}^{9} f(\bar{x}_k, \bar{y}_k)\ \Delta\ A_k$, assuming that $(\bar{x}_k, \bar{y}_k)$ are the centers of the nine squares. Take $f(x,y)=6+2\mathrm{x}+3\mathrm{y}$.

Answer 3: ____________________________________

4. (25 points) Calculate the surface area of the part of the saddle $az = x^2 - y^2$ inside the cylinder $x^2 + y^2 = a^2$, $a > 0$.

Answer 4: ____

5. (20 points) Rewrite the integral $\int_0^2 \int_0^{9-x^2} \int_0^{2-x} f(x, y, z)\, dz\, dy\, dx$ changing the order of integration to $dz\, dx\, dy$.

Answer 5: ____

6. (25 points) Evaluate the integral $\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{-\sqrt{9-x^2-y^2}}^{\sqrt{9-x^2-y^2}} (x^2+y^2+z^2)^{3/2}\, dz\, dy\, dx$.

Hint – Convert to spherical coordinates.

Answer 6: ____

7. Find the divergence and curl of the following vector fields. (15 points each)

a) $\boldsymbol{F}(x, y, z) = x^2\boldsymbol{i} + y^2\boldsymbol{j} + z^2\boldsymbol{k}$

Divergence 7a):______________________________

Curl 7a):______________________________

b) $\boldsymbol{F}(x, y, z) = yz\,\boldsymbol{i} + xz\,\boldsymbol{j} + xy\,\boldsymbol{k}$

Divergence 7b):______________________________

Curl 7b):______________________________

8. (25 points) Prove that if the curl of a vector field is not zero, then the vector field is not conservative.

Extra Credit (20 points): Prove the identity:

$$\int_0^x \int_0^v \int_0^u f(t)\,dt\,du\,dv = (1/2)\int_0^x (x-t)^2 f(t)\,dt$$