

Name \_\_\_\_\_ Date \_\_\_\_\_

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. (20 pts) Find the directional derivative of  $f(x, y) = x^2 - 3xy + 2y^2$  at  $\mathbf{p} = (-1, 2)$  in the direction of  $\mathbf{a} = 2\mathbf{i} - \mathbf{j}$ .

Answer: \_\_\_\_\_

2. (25 pts) For the surface  $x^3y^2 - yz^2 + x^2y^3z - 5x + 2 = 7$  find the equation of the tangent plane at the point  $(1,1,1)$ .

Tangent Plane: \_\_\_\_\_

3. (8 pts) Draw the set  $S = \{(x, y) \mid x \in (-2, 2) \wedge y \in [-3, 3]\}$  .

(5 points) Is the set open, closed, or neither? \_\_\_\_\_

(3 points ) Is the point (0,-3) an interior point? \_\_\_\_\_

(3 points) Is the point (2,-3) an interior point? \_\_\_\_\_

(3 points) Is the point (0,-3) a boundary point? \_\_\_\_\_

(3 points) Is the point (2,-3) a boundary point? \_\_\_\_\_

4. (20 pts) Find all critical points for  $f(x, y) = x^3 + y^3 - 6xy$  . Determine whether each point is a minimum, maximum or saddle point.

Critical point(s) (Specify whether they're min, max or saddle.):

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5. For  $z=f(x,y)=x^2+y^2+e^{x+y}$ , find

(a) (10 pts)  $\frac{\partial z}{\partial y}$  at (1, 1)

(b) (10 pts)  $f_{xy}$

Answer : \_\_\_\_\_

Answer : \_\_\_\_\_

6. (20 points) Find the minimum of  $x^2 + 4xy + y$  subject to the constraint  $x - y - 6 = 0$  .

Answer: \_\_\_\_\_

7. (25 points) Find the dimensions of the rectangular box of volume  $V_0$  for which the sum of the edge lengths is least.

Answer : \_\_\_\_\_

8. (20 pts) Find  $\frac{dw}{dt}$  if  $w(x, y) = x^2 y^3$ ,  $x = t^3$ ,  $y = t^2$ . (Your answer must be only in terms of  $t$ .)

Answer : \_\_\_\_\_



9. Find the limit, if it exists. (Show all your reasoning.)

(a)  $\lim_{(x,y) \rightarrow (1,1)} \frac{x - 3x^2 + 3xy - y^2}{y - 2x^2}$  (10 pts)

Answer : \_\_\_\_\_

(b)  $\lim_{(x,y) \rightarrow (0,0)} \frac{\tan(x^2 + y^2)}{\sqrt{x^2 + y^2}}$  (15 pts)

Answer : \_\_\_\_\_