Math2210 Midterm 1 Summer, 2012 Dylan Zwick

Name ______________________________ Date _____________

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. For $x=2\sqrt{t-2}$ and $y=3\sqrt{4-t}$ such that $2 \leq t \leq 4$, do the following:

(a) (10 pts) Eliminate the parameter to obtain the corresponding Cartesian equation.

Answer 1(a):____

(b) (10 pts) Graph the curve.

(c) (5 pts) Indicate if the curve is simple and/or closed.

Simple: T or F (circle one)

Closed: T or F (circle one)

2. (10 pts) Find the length of the curve given by $x=4\sqrt{t}$ and $y=t^2+\frac{1}{2t}$ for $\frac{1}{4}\le t\le 1$.

Answer 2: ____________________

3. (15 pts) For position vector given by $\boldsymbol{r}(t)=\sin 2t\,\boldsymbol{i}+\cos 3t\,\boldsymbol{j}+\cos 4t\,\boldsymbol{k}$, find the velocity and acceleration vectors and the speed at $t=\frac{\pi}{4}$.

$\boldsymbol{v}(t)$ = ____________________

$\boldsymbol{a}(t)$ = ____________________

speed at $t=\frac{\pi}{4}$ = ____________________

4. (10 pts) Find the limit, if it exists. $\lim_{t \to 0^+} \left[\ln(t^3)\,\boldsymbol{i} + t^2 \ln(t)\,\boldsymbol{j} + t\,\boldsymbol{k} \right]$

Answer (4) : ______________________________

5. (10 pts) Find the equation of the sphere that has the line segment joining (3, 1, 7) and (7, 5, 5) as a diameter.

Radius = ________________________

center = ________________________

Eqn of sphere: ____

6. (10 pts each) Let $\boldsymbol{a}=\langle 4,1,2\rangle$, $\boldsymbol{b}=\langle 2,4,1\rangle$ and $\boldsymbol{c}=6\boldsymbol{i}+3\boldsymbol{j}$. Find each of the following.

(a) $2\boldsymbol{a}-3\boldsymbol{c}$

$2\boldsymbol{a}-3\boldsymbol{c}$ = ______________________________

(b) $\boldsymbol{a}\cdot(\boldsymbol{b}+\boldsymbol{c})$

$\boldsymbol{a}\cdot(\boldsymbol{b}+\boldsymbol{c})$ = ______________________________

(c) $\boldsymbol{b}\cdot\boldsymbol{c}-|\boldsymbol{b}|$

$\boldsymbol{b}\cdot\boldsymbol{c}-|\boldsymbol{b}|$ = ______________________________

(Note: This is # 6 continued $\boldsymbol{a}=\langle 4,1,2\rangle$, $\boldsymbol{b}=\langle 2,4,1\rangle$ and $\boldsymbol{c}=6\boldsymbol{i}+3\boldsymbol{j}$

(d) $\hat{\boldsymbol{c}}$ (the unit vector)

$\hat{\boldsymbol{c}}$ = ______________________________

(e) $\boldsymbol{a}\times(\boldsymbol{b}\times\boldsymbol{c})$

$\boldsymbol{a}\times(\boldsymbol{b}\times\boldsymbol{c})$ = ______________________________

(f) $\boldsymbol{a}\cdot(\boldsymbol{b}\times\boldsymbol{c})$

$\boldsymbol{a}\cdot(\boldsymbol{b}\times\boldsymbol{c})$ = ______________________________

7. (10 pts each) For $\boldsymbol{a} = 3\boldsymbol{i} + 4\boldsymbol{j} + 5\boldsymbol{k}$ and $\boldsymbol{b} = 2\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k}$, find each of the following:

(a) Direction cosines for $\boldsymbol{a}$.

$\cos\alpha$ = ____________________

$\cos\beta$ = ____________________

$\cos\gamma$ = ____________________

(b) The angle θ between $\boldsymbol{a}$ and $\boldsymbol{b}$. (Just write a simplified expression. If you don't have a calculator just write the numerical formula for the angle.)

θ = ______________________________

(c) Find the projection of $\boldsymbol{b}$ onto $\boldsymbol{a}$.

Projection of $\boldsymbol{b}$ onto $\boldsymbol{a}$ = ______________________________

8. (10 pts each) For the planes given by

$$3x - 2y + 5z = 16$$

and

$$4x + 2y + z = 13,$$

answer the following questions.

(a) Find the line of intersection between the planes and write that line in parametric equations.

Line: ____

(b) Find the equation of the plane that is perpendicular to the line of intersection and goes through the point (1, 3, 2).

Equation of plane: ____

9. (a) (10 pts) Convert $x^2 + y^2 = 2y + 2x$ from a Cartesian coordinate equation into an equation in cylindrical coordinates.

Answer : ____

(b) (10 pts) Convert $r = 2\sin\theta$ from a cylindrical coordinate equation into an equation in Cartesian coordinates.

Answer : ______________________________________