Math2210 Final Exam Summer, 2012 Dylan Zwick

Name ______________________________________ Date _____________

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. *(15 points)* For position vector given by $\boldsymbol{r}(t) = (t^3 - 2t^2 + 5t)\boldsymbol{i} + (t^2 + t + 1)\boldsymbol{j}$, find the velocity and acceleration vectors and the speed at $t = 1$.

$\boldsymbol{v}(t)$ (6 points) = ____

$\boldsymbol{a}(t)$ (6 points) = ____

speed at $t = 1$ (3 points) = ______________________

2. *(15 points)* Let $\boldsymbol{a} = \langle 2,-1,3 \rangle$, $\boldsymbol{b} = \langle 1,3,2 \rangle$ and $\boldsymbol{c} = \langle 0,1,-1 \rangle$. Find each of the following.

(a) $2\boldsymbol{a} - 3\boldsymbol{c}$ (3 points)

$2\boldsymbol{a} - 3\boldsymbol{c}$ = ______________________________

(b) $\boldsymbol{a} \cdot (\boldsymbol{b} + \boldsymbol{c})$ (3 points)

$\boldsymbol{a} \cdot (\boldsymbol{b} + \boldsymbol{c})$ = ______________________________

(c) projection of $\boldsymbol{a}$ onto $\boldsymbol{b}$ (6 points)

projection of $\boldsymbol{a}$ onto $\boldsymbol{b}$ = ______________________________

(d) $\hat{\boldsymbol{a}}$ (the unit vector) (3 points)

$\hat{\boldsymbol{a}}$ = ____________________________________

3. (20 points) For the points $A(1, 3, 2)$, $B(0, 3, 0)$ and $C(2, 4, 3)$

(a) *(10 points)* Find a normal vector to, and equation for, the plane through points A, B and C.

Normal vector = ________________________

Equation of plane: ____

(b) *(10 points)* Write a set of parametric equations for the line through point B and perpendicular to the plane in part (a).

Line: ____

4. *(20 points)* Find the directional derivative of $f(x, y, z) = x^2 y + y^2 z + z^2 x$ at $\mathbf{p} = (1, 0, 1)$ in the direction of $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$.

Answer: ____

5. *(20 points)* Find all critical points of the function $f(x, y) = xy^2 - 6x^2 - 3y^2$.
Determine if each critical point is a local minimum, a local max, or neither. If there are none of the given type of point, just write "None".

Critical Points: ______________________________

Local Max: ______________________
Local Min: ______________________
Neither: ______________________

6. (20 points) Calculate the integral

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx$$

Note – You must provide a correct calculation. Don't just state the answer.

Answer:___________________________________

7. (20 points) Calculate the surface area of the part of the elliptic paraboloid $z = 16 - x^2 - y^2$ above the xy-plane.

Surface Area:____

8. (30 points) Calculate the following integrals:

a) $\int_0^{\frac{\pi}{2}} \int_0^{\sin y} e^x \cos y \, dx \, dy$ (10 points)

Answer:____

b) $\int_0^{\frac{\pi}{2}} \int_0^{z} \int_0^{y} \sin(x+y+z) \, dx \, dy \, dz$ (10 points)

Answer:____

c) $\int_0^2 \int_x^2 6x e^{y^3} \, dy \, dx$ (10 points)

Answer:____

9. *(20 points)* Given $\boldsymbol{F}(x, y, z) = 5x^3 y z \, \boldsymbol{i} - 2y x^2 \, \boldsymbol{j} + y^3 z^2 \, \boldsymbol{k}$, calculate the following.

(a) div $\boldsymbol{F}$ (5 points)

div $\boldsymbol{F}$ = ____

(b) curl $\boldsymbol{F}$ (5 points)

curl $\boldsymbol{F}$ = ______________________________

(c) $\nabla(\nabla \cdot \boldsymbol{F})$ (5 points)

$\nabla(\nabla \cdot \boldsymbol{F})$ = ______________________________

(d) $\nabla \cdot (\nabla \times \boldsymbol{F})$ (5 points)

$\nabla \cdot (\nabla \times \boldsymbol{F})$ = ______________________________

10. (30 points)

a) (20 points) Determine if the field $\boldsymbol{F} = (2xy + z^2)\boldsymbol{i} + x^2\boldsymbol{j} + (2xz + \pi\cos\pi z)\boldsymbol{k}$ is conservative. If it is conservative, find a function f such that $\nabla(f) = \boldsymbol{F}$.

Conservative? (Circle One) True False

f (if it exists) = ______________________________

(This is problem 10 continued, so $\boldsymbol{F} = (2xy + z^2)\boldsymbol{i} + x^2\boldsymbol{j} + (2xz + \pi\cos\pi z)\boldsymbol{k}$).

b) (10 points) Calculate the line integral $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$ where C is the line segment connecting the points (0,0,0) and (1,2,3).

Answer:______________________________________

11. (20 points) Calculate the line integral $\oint_C xy\,dx + (x+y)\,dy$ where C is the triangle with vertices (0,0), (2,0), and (0,1).

Hint – Use Green's theorem.

Answer:________________________________

12. (20 points) Calculate the surface integral $\int\int_G g(x,y,z)\,dS$ where $g = x + y$ and G is the part of $z = \sqrt{4 - x^2}$ with 0 ≤ x ≤ $\sqrt{3}$ and 0 ≤ y ≤ 1.

Note - $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right)$.

Answer: ______________________________________

Extra Credit:

(10 points) What did you think of the class, and what could be done to improve it?