

Name Solutions Date 7/27/2010

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. (10 points each) Evaluate the following integrals.

(a) $\int_1^2 \int_0^3 (xy + y^2) dx dy$

$$= \int_1^2 \left(\frac{x^2 y}{2} + xy^2 \right) \Big|_0^3 dy$$

$$= \int_1^2 \left(\frac{9y}{2} + 3y^2 \right) dy = \frac{9y^2}{4} + y^3 \Big|_1^2$$

$$= (9+8) - \left(\frac{9}{4} + 1 \right) = 17 - \frac{13}{4} = \frac{68}{4} - \frac{13}{4} = \frac{55}{4}$$

Answer: $\boxed{\frac{55}{4}}$

(b) $\int_{-1}^1 \int_0^1 x e^{x^2} dx dy$

$$u = x^2 \quad \frac{du}{2} = x dx$$

$$= \int_{-1}^1 \int_0^1 \frac{e^u}{2} du dy$$

$$= \int_{-1}^1 \left(\frac{e^u}{2} \Big|_0^1 \right) dy = \int_{-1}^1 \left(\frac{e-1}{2} \right) dy$$

$$= \left(\frac{e-1}{2} \right) y \Big|_{-1}^1 = \boxed{e-1}$$

Answer: $\boxed{e-1}$

(c) $\int_0^2 \int_{-x}^x e^{-x^2} dy dx$.

$$= \int_0^2 y e^{-x^2} \Big|_{-x}^x dx$$

$$= \int_0^2 2x e^{-x^2} dx \quad \begin{array}{l} u = x^2 \\ du = 2x dx \end{array}$$

$$= \int_0^4 e^{-u} = -e^{-u} \Big|_0^4$$

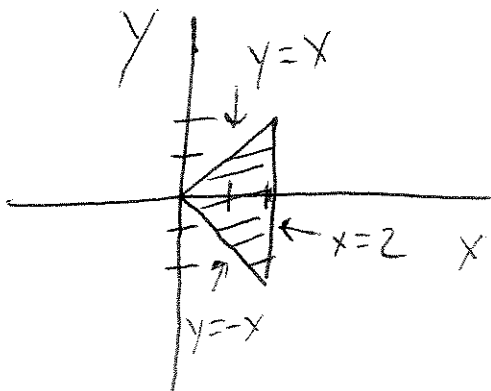
$$= -e^{-4} - (-1)$$

$$= 1 - e^{-4}$$

Answer: _____

$$\boxed{1 - e^{-4}}$$

2. (10 points) Graph the domain of integration for part c) above, and write the integral with the order of integration switched. Don't bother evaluating the integral again.



$$= \int_{-2}^2 \int_{|y|}^2 e^{-x^2} dx dy$$

$$= \int_{-2}^0 \int_{-y}^2 e^{-x^2} dx dy$$

$$+ \int_0^2 \int_y^2 e^{-x^2} dx dy$$

Answer: _____